

Figure III-9. Plot of RPM, Disk Spacing, and Torque Versus Outer Rotor Radius for Liquid Hydrogen for Head = 200 Feet

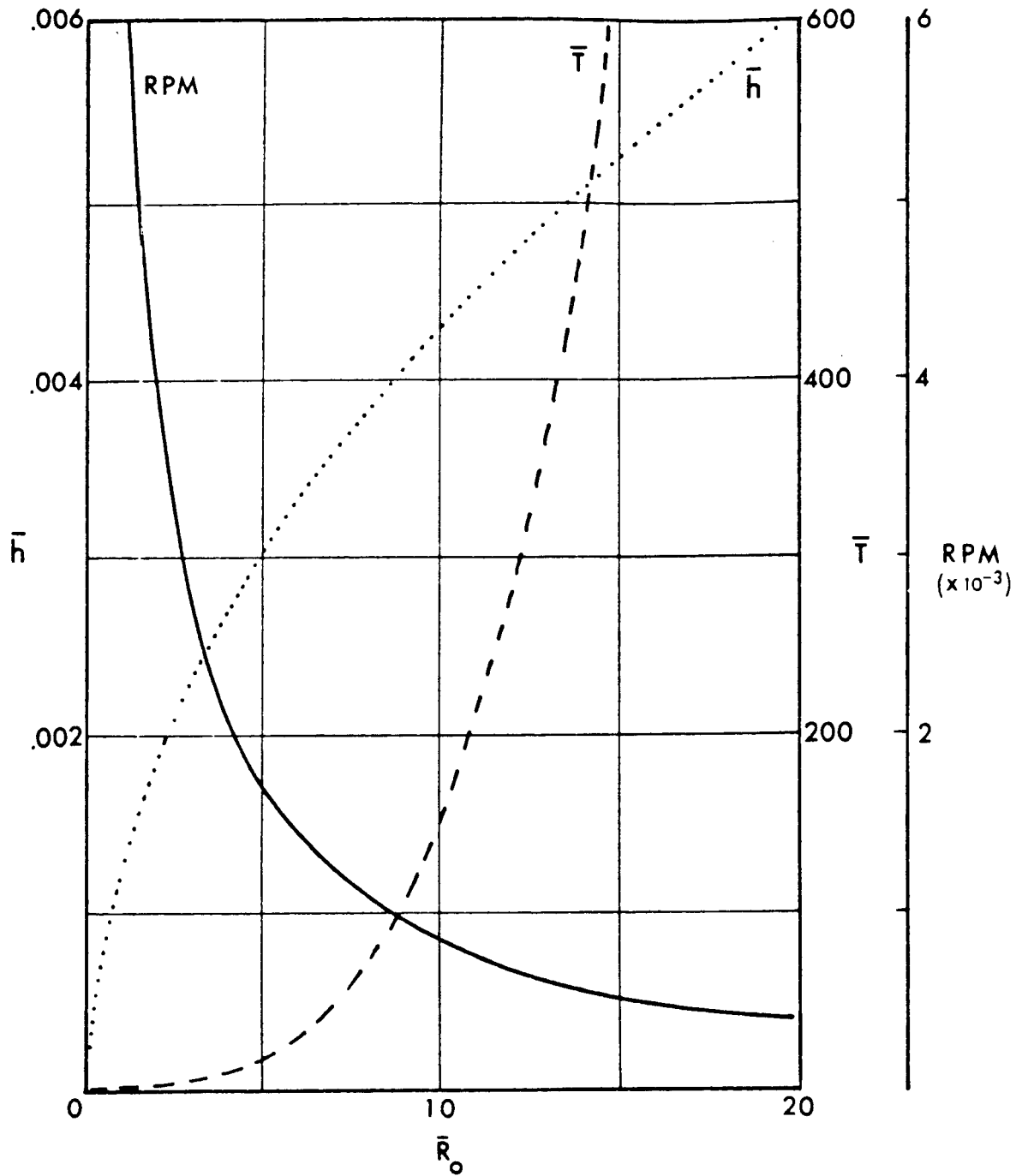


Figure III-10. Plot of RPM, Disk Spacing, and Torque Versus Outer Rotor Radius for Liquid Sodium for Head = 200 Feet

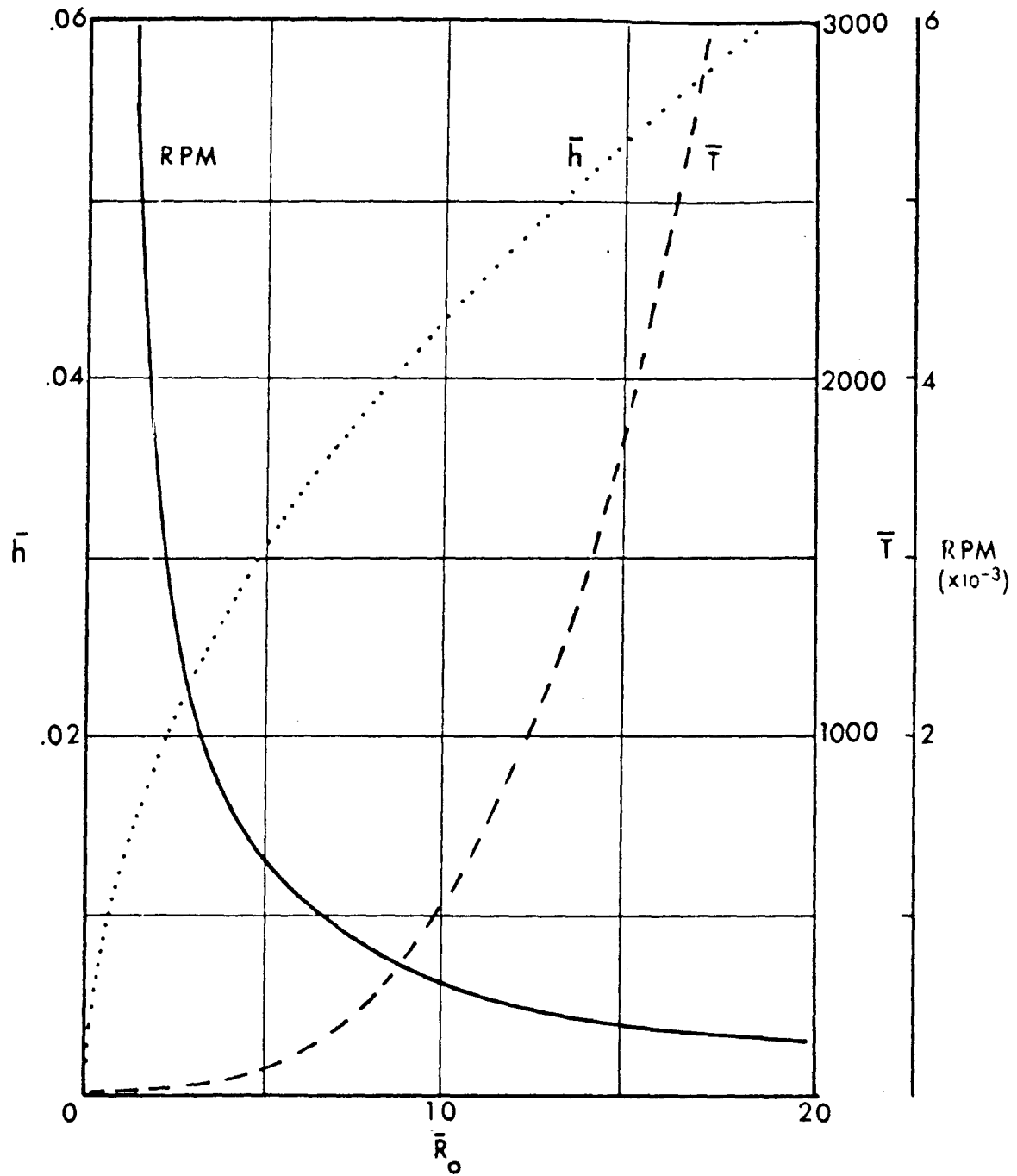


Figure III-11. Plot of RPM, Disk Spacing, and Torque Versus Outer Rotor Radius for Glycerine for Head = 200 Feet

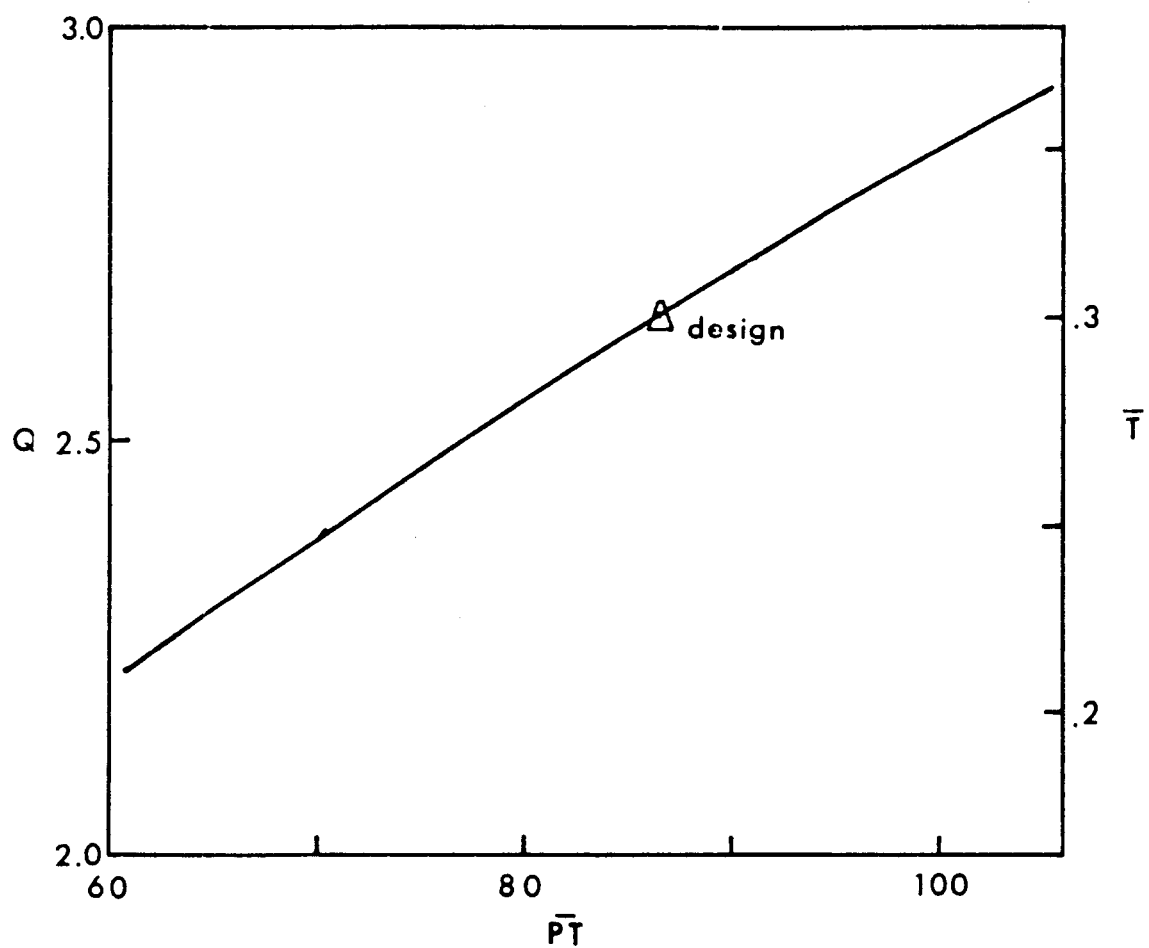


Figure III-12. Illustration of Off-Design-Point Operation for Liquid Hydrogen. Design $\bar{R}_0 = 2.0$ Inches and Head = 200 Feet

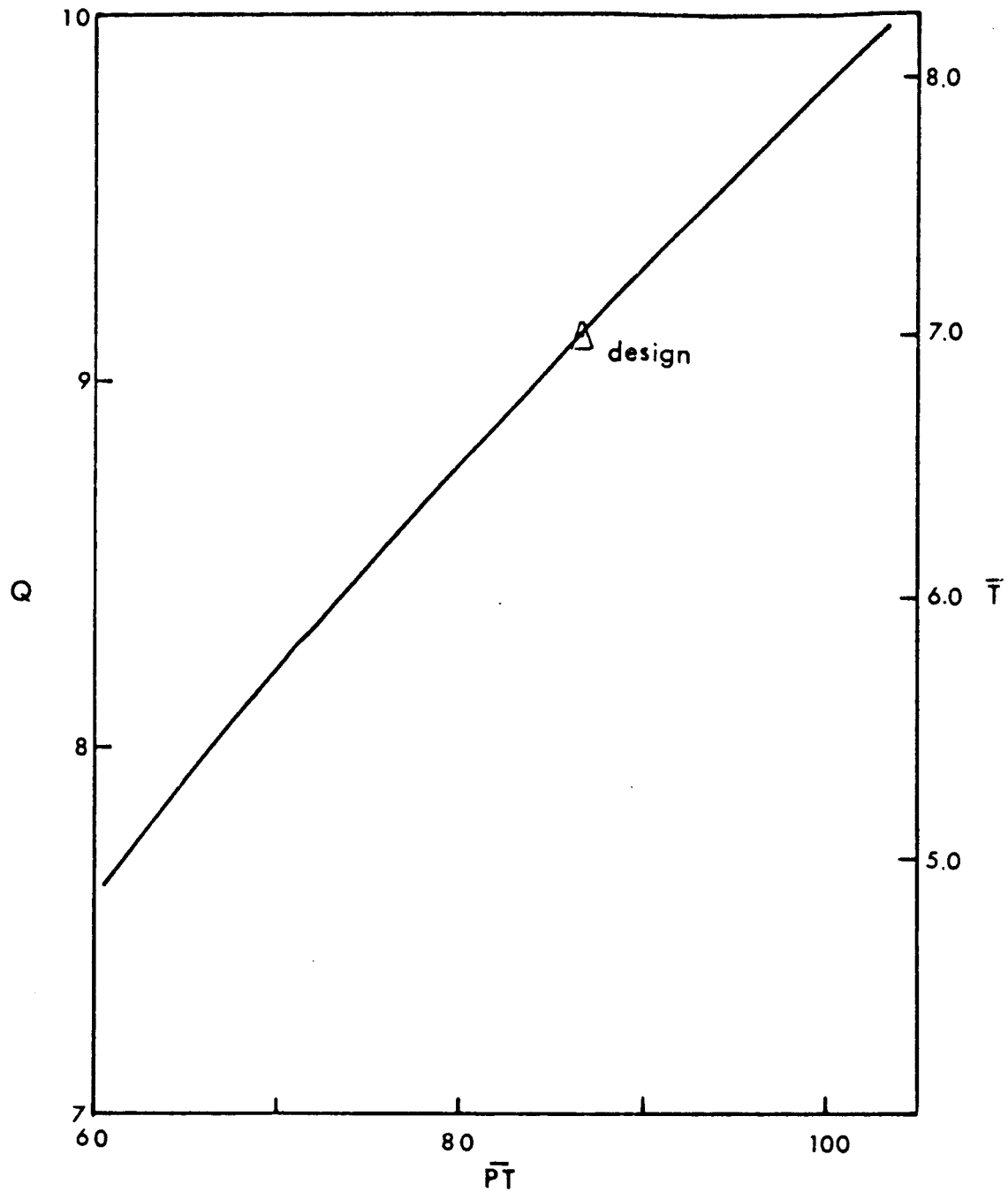


Figure III-13. Illustration of Off-Design-Point Operation for Liquid Sodium. Design $\bar{R}_0 = 4.0$ Inches and Head = 200 Feet

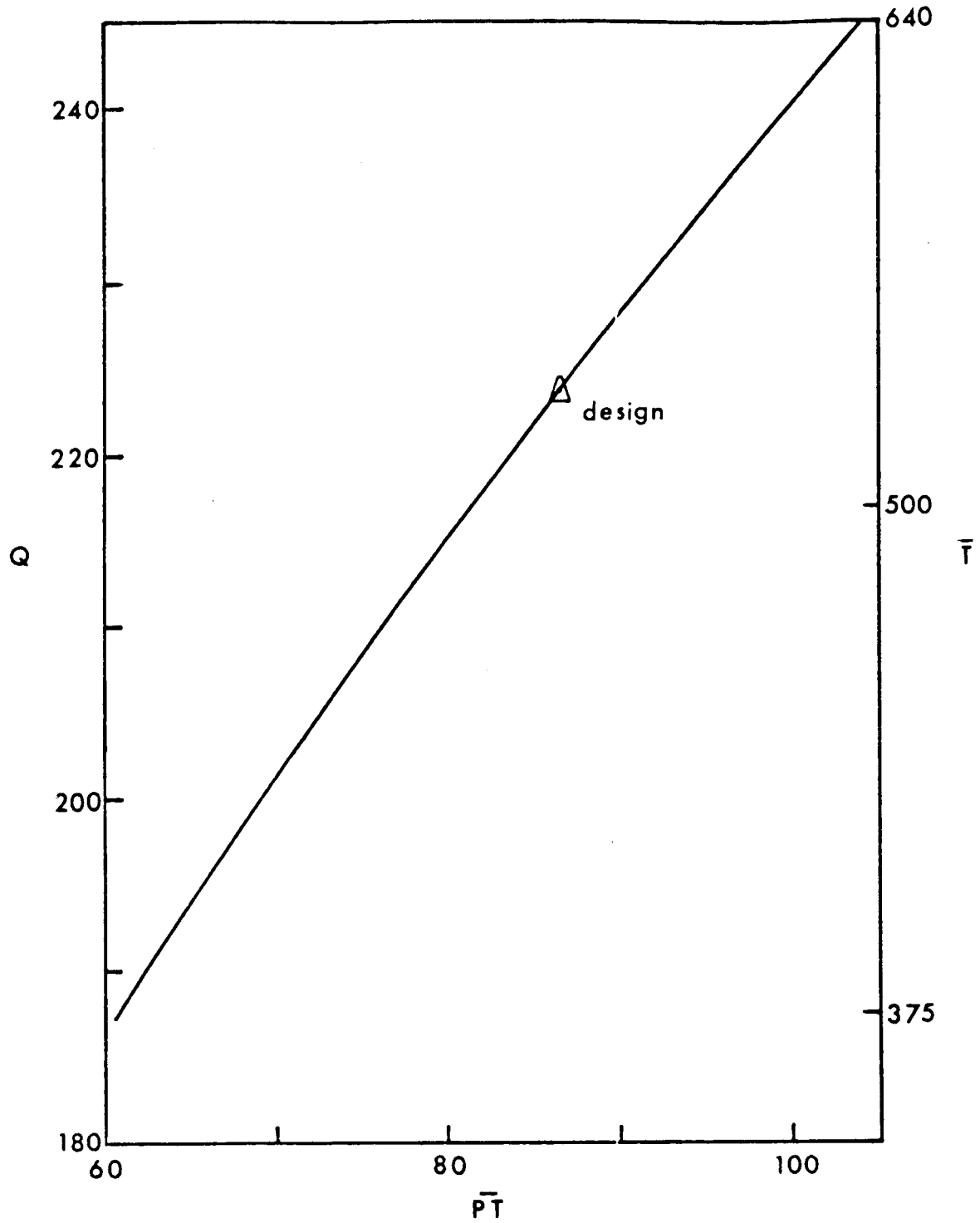


Figure III-14. Illustration of Off-Design-Point Operation for Glycerine. Design $\bar{R}_0 = 10.0$ Inches and Head = 200 Feet

Chapter IV

CONCLUSION

SUMMARY

The results of the Boyack analysis were used to plot performance maps for the multiple-disk turbine. The Boyack solution is applicable for the conditions of a three-dimensional, laminar flow, of an incompressible Newtonian fluid, in radially inward flow between parallel, co-rotating disks. Parabolic and uniform input velocity profiles, with full admission of the working fluid to the rotor, were assumed. Performance data for the general flow problem required the three dimensionless input parameters be specified, N_{RE} , U_o and V_o . In this investigation N_{RE} , U_o , and V_o were allowed to vary over the ranges of $0.5 < N_{RE} < 10.0$, $0.01 < U_o < 1.0$, and $0.8 < V_o < 1.3$. The performance parameters, η , T , and PT , at specified exit radii were plotted individually with respect to Reynolds number, N_{RE} , and radial input velocity, U_o .

The performance maps in Appendix C enable a turbine designer to determine maximum limits of efficiency and performance for a multiple-disk turbine. Having used the performance curves to determine a particular area of interest, a designer could then refer to the output sheets presented in Appendix B to obtain more precise data for calculating the turbine operating specifications. The dimensional

characteristics such as disk spacing, exhaust radius, and nozzle angle can then be determined by using a design program similar to the one presented in Appendix D.

RECOMMENDATIONS

The constant characteristic values on the performance maps were hand plotted using the output data from Boyack's analysis. The process was very time consuming. It is recommended that a digital computer program be constructed, using the output data on the summary sheets in punched card form, to make initial plots of U_0 versus η , T , and PT with N_{RE} , V_0 , and R_i constant. A "parabolic fit" technique can be used to make the plots, since the initial curves are smooth and without inflections. The results could be graphical or tabular depending on the peripheral equipment available.

A more sophisticated program could be developed to do the cross plotting by interpolating a series of initial output curves. The output could be graphical or tabular again depending on the peripheral equipment available. This approach would result in greater accuracy and a more efficient method of plotting performance maps.

FURTHER STUDY

Performance curves that are produced for conventional turbines, as in references [27] and [28], are expressed in terms of specific diameter versus specific speed, where dimensional parameters such as pressure head, volume flow rate, speed, and diameter are combined to

form the specific parameters. Further investigation into the derivation of similar expressions for the multiple-disk turbine would be beneficial. The output data shown in the performance maps in Appendix C could be represented by a single set of performance curves rather than three individual sets.

An additional area for investigation is the region where $U_o > 1.0$, and all V_o and N_{RE} . With these input parameters the output data indicates that there is pumping action in the outer periphery of the rotor. The possibility of a radial inflow pump exists in this area of operation.

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APPENDIX A

MODIFIED BOYACK PROGRAM

PROGRAM PUMP (INPUT,OUTPUT)

PROGRAM PUMP, M CRAWFORD, ARIZONA STATE UNIVERSITY, 1971
PROGRAM UPDATED FOR CDC 6400, SCOPE 3.3

THIS IS A MODIFICATION OF THE TURBINE PROGRAM, YIELDING
MAGNITUDE (NON-DIMENSIONALIZED BY OUTER RADIUS * OMEGA)
AND DIRECTION (DEGREES FROM A RADIAL LINE) OF VECTOR AVERAGE
VELOCITY AT SELECTED RADIAL STATIONS, BY MEANS OF SUBROUTINE
VECTOR.

THIS PROGRAM TREATS THE LAMINAR FLOW OF AN INCOMPRESSIBLE,
NEWTONIAN FLUID, WITH RADially INWARD OR OUTWARD THROUGH-
FLOW, BETWEEN PARALLEL, CO-ROTATING DISKS.
THE SOLUTION METHOD CONTAINS A FINITE-DIFFERENCE SOLUTION SEG-
MENT AND AN INTEGRAL SOLUTION SEGMENT.
INPUT VELOCITY PROFILE SHAPES MAY BE PARABOLIC, UNIFORM, MATSCH
ASYMPTOTIC, OR ARBITRARY SPECIFIED.
THE F-D SEGMENT IS USED AS A START-UP FOR UNIFORM INPUT PROFILES,
WITH MATCHUP TO INTEGRAL METHOD UPON SATISFACTION OF MATCHUP
CRITERIA.
POLYS OF 4,6,8 ORDER MAY BE SPECIFIED FOR INTEGRAL SEGMENT. FOR
HIGHER ORDER PROFILES, REPLACE SUBROUTINE DERV. THE INTEGRAL
SEGMENT ALLOWS ORDER REDUCTION TO 4 OR 6 DURING INTEGRATION.

PARAMETER *****	TYPE ****	DESCRIPTION *****	COLUMNS *****
CARD 1 *****			
LIMP	INT	NUMBER INFO STORAGE POINTS	1-2
CARD 2 *****			
XPNT (I)	FP	STORAGE POINTS	4F15.0
CARD 3 (UNIFORM PROFILE CONTROL) *****			
KK1	INT	NO. F-D STEPS BEFORE MATCHUP TRY	1-5
KK2	INT	INTERMED. PRINT INTERVAL, WITHIN KK1	6-10
KK3	INT	KK3=0 NO PRINT AT KK2 INTERVAL KK3=1 PRINT	11-15
PCTU	FP	MAX PERCT DEV, INTEGRATED F-D PRO- FILE TO POLY CURVE-FIT (U VEL)	16-30
PCTV	FP	MAX PERCT DEV, V VEL F-D POINT TO POLY CURVE-FIT POINT	31-45
NOTE -- IF COMPARISISON TESTS FAIL, F-D SEGMENT CONTINUES WITH MATCHUP ATTEMPTS AT 10 F-D INTERVALS			
CARD 4 *****			
NOPNT	INT	NOPNT=0 PRINT EVERY XPNT INTERVAL NOPNT=1 PRINT ONLY SUMMARY SHEET	1-5

C		MODE	INT	MODE=4	6TH ORDER POLY	6-10
C				MODE=6	6TH ORDER POLY	
C				MODE=8	8TH ORDER POLY	
C		INPRO	INT	INPRO=1	PARABOLIC INPUT PROFILE	11-15
C				INPRO=2	UNIFORM INPUT PROFILE	
C				INPRO=3	MATSCH ASYMPT PROFILE	
C				INPRO=4	ARBITRARY PROFILE	
C		KCMODE	INT	KCMODE=0	NO PROFILE ORDER REDUCE	16-20
C				KCMODE=4	4TH ORDER REDUCE	
C				KCMODE=6	6TH ORDER REDUCTION	
C		XFIT	FP		INTERVAL FOR REDUCTION IN POLY	21-35

CARD 5

C		U0,V0,PE	FP	PARAMETERS FOR INPRO=1,2	3F15.0
C	OR				
C		U0,PE	FP	PARAMETERS FOR INPRO=3	2F15.0
C	OR				
C		AUP(I,J)	FP	Z COORD, U COMP, V COMP ARRAY, (MODE/2+1) PTS REQD	3F15.0

NOTE -- DATA CARDS 4,5 REPEAT AS NEW DATA. USE ONE BLANK
CARD TO TERMINATE PROGRAM

* * * * *

DIMENSIONS FOR 12TH ORDER VELOCITY PROFILES AND 31 PRINT STATIONS
WITH PROFILE VALUES AT 21 VALUES OF 0.025 SPACING

TO COMPUTE ABOVE 8TH ORDER, SUBROUTINE DERV MUST BE REPLACED
WITH THE ARBITRARY ORDER PROFILE DERV SUBROUTINE

C	COMMON	QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
1		Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
2		Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
3		X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
4		M3,M4,MT,NT,RE,U0,V0,DPDR,P,P1,P2,P3,P4,PG1,PG2,
5		MODE,NOPNT,INPRO,KKONT,MDIOG,KF,KFF,KPNT,SIG,ALPRM,
6		ATES(64,65),XTES(64),
7		RSV(31),ETASV(31),PTSV(31),PSV(31),TQUESV(31),
8		DPDRSV(31),ACD(13),ZINF(13),ZVUV(21,13),ZVW(21,13),
9		INUM(31),RINF(11),AUP(7,3),UW(21),VW(21),RI1,
1		ZW(21),WW(21),Q(13,13),XPNT(31),RI2,IS,LU,NTP,PI,NTH,
2		VMAG(31),VANG(31)

DIMENSION IFIT(7)
EQUIVALENCE (X,R)

C	*1*	READ NUMBER OF PRINT AND/OR DATA STORAGE STATIONS READ 7000, LIMP
C	*2*	READ STORAGE STATION ARRAY READ 7005, (XPNT(I), I=1,LIMP)
C	*3*	READ UNIFORM PROFILE CONTROL DATA READ 7015, KK1,KK2,KK3,PCTU,PCTV
C	*4*	READ PRINT CONTROL,PROFILE ORDER,PROFILE SHAPE,REDUCTION ORDER,RED.PT

```

1 READ 101, NOPNT,MODE,INPRO,KCMODE,XFIT
  IF (MODE.EQ.0) CALL EXIT

C
  GO TO (205,205,210,215), INPRO

C
*5*   PARABOLIC INLET PROFILES
C     OR
C     UNIFORM INLET PROFILES
C
205 READ 102, U0,V0,RE
  GO TO 220

C
*5*   READ MATSCH ASYMPTOTIC PROFILES
C
210 READ 103, U0,RE
  GO TO 220

C
*5*   READ ARBITRARY PROFILES AS SETS OF Z COORD,U VEL COMP,V VEL
C     COMP FOR (MODE/2+1) POINTS, INCLUDING TWO BOUNDARY POINTS
C     AUP(I,1)=Z COORD  AUP(I,2)=U COMP  AUP(I,3)=V COMP
C
215 READ 104, RE
  LIM=MODE/2+1
  READ 105, ((AUP(I,J), J=1,3), I=1,LIM)
220 CONTINUE
  IUNIFM=0
  IF (INPRO.EQ.2) IUNIFM=1
C
  CURVE FIT POINTS
  CALL CFIT (IFIT,MODE)

C
C$$$$ CDC 6400 TIME RECORD
  RTIME=SECOND(A)

C
  XI   INITIAL INTEGRATION POINT
  XI=XPNT(1)
C
  FINAL CUT-OFF VALUE INDEPENDENT VARIABLE
  XC=XPNT(LIMP)
C
  INITIAL PRESSURE
  PINL=0.

C
  SIG=+1.
  IF (XPNT(2).LT.XPNT(1)) SIG=-1.
  XWS=XI
  NT=MODE
  NTSV=MODE
  NTP=NT+1
  NTH=NT/2
  KCMODD=KCMODE
  IS=0
  PI=3.14159265
C
  *   *   *   *   *   *   *   *   *   *   *   *
C
  DIAGNOSTIC FLAGS
C
  MDIOG=0   PRINT EVERY DOUBLE OR HALVE OF STEP SIZE
  MDIOG=1   NO PRINT
C
  MDIOG=1

C
  MP=0     PRINT EVERY INTEGRATION STEP

```



```

390 KONT=1
    MC=0
    MT=0
    M1=0
    M2=0
    M3=0
    M4=0
    MCO=0
    KF=0
    KFF=0
    EF=1.0E11
    DXM=SIG *0.0025
    DXTER=1.E-12
    EMAX=1.0E-04
    EMIN=EMAX/100000.0
    X=XI
    DX=DXI
    P=PINL
C  OBTAIN ORIGINAL VALUES OF DEPENDENT VARIABLES
    DO 4 I=1,NTP
    AC(I)=0.0
    BC(I)=0.0
    ACD(I)=0.0
    CONTINUE
    IF (INPRO.NE.2) GO TO 8
    UVPR=UO*VO
    UUVV=UO*UO*UO+UO*VO*VO
    RI1=59.*UVPR*0.025/3.
    RI2=59.*UUUVV*0.025/3.
    IF (SIG.GT.0.) PINL=-RI2/UO
    P=PINL
    IS=1
    XWS=XPNT(2)
    INUM(1)=0
    PSV(1)=PINL
    RSV(1)=XI
    ETASV(1)=0.
    PTSV(1)=RI2/UO+PINL
    TQUESV(1)=0.
8  CONTINUE
    KER=0
    CALL      INPUT(KER,AUP,R,IFIT ,RE,UO,VO,MODE,INPRO,SIG,P,LU,
1  KK1,KK2,KK3,PCTU,PCTV)
    IF (KER.EQ.2) GO TO 50
    IF (KER.EQ.3) GO TO 51
    DO 6 I=1,NTH
    J=I*2-1
    K=NTH+I
    YI(I)=AC(J)
    YI(K)=BC(J)
6  CONTINUE
    DO 16 I=1,NT
    Y(I)=YI(I)
    E(I)=0.
16 CONTINUE

```

```

IF (INPRO.EQ.2.OR.ILIM.NE.0) GO TO 888
INITIAL TORQUE AND DYNAMIC PRESSURE
C RI1=0.0
RI2=0.0
DO 18 I=1,NTP,2
DO 18 J=1,NTP,2
RI1=RI1+AC(I)*BC(J)/((I+J-1)*2.0**(I+J-1))
DO 18 K=1,NTP,2
RI2=RI2+(AC(I)*AC(J)*AC(K)+AC(I)*BC(J)*BC(K))/((I+J+K-2)*
12.0**(I+J+K-2))
18 CONTINUE
888 CONTINUE
CALL DERV
CALL PRNT(1)
KPAR=1
IF (INPRO.EQ.2) KPAR=0
CALL PROFIL (KPAR,XWS)
20 CALL MOVE
KKONT=KKONT+1
IF (KKONT.GT.KKONTM) GO TO 55
KPNT=KPNT+1
IF (MT) 22,21,22
21 KONT=KONT+1
IF(KONT-4) 22,22,23
22 CALL RKGI
DO 24 I=1,NT
E(I)=0.0
24 CONTINUE
IF (MDIOG.EQ.0) CALL PRNT(2)
GO TO 25
23 CALL ADMI
IF (Z2(1)-EF) 25,25,40
25 IF (MP.EQ.0) CALL PRNT(2)
IF (ABS(DX).LT.DXTER) GO TO 32
C EXAMINE U PROFILE FOR INFLECTION
UINFL=0.0
DO 26 I=3,NTP,2
UINFL=UINFL+(I-1)*AC(I)*ZINF(I)
26 CONTINUE
IF (SIG *UINFL) 27,27,28
27 IF (INFL.EQ.0) GO TO 29
INFL=0
ICONT=ICONT+1
RINF (ICONT)=R
GO TO 29
28 IF (INFL.EQ.1) GO TO 29
INFLT=1
ICONT=ICONT+1
RINF(ICONT)=R
INFL=1
29 CONTINUE
CALL ITRO (MCO)
IF (MCO.EQ.1) GO TO 601
IF ((ABS(X-XWS)-ABS(0.99*DX)).GE.0.) GO TO 602
601 KPAR=1

```

```

GO TO 603
602 IF (KPNT.NE.KPMAX) GO TO 599
    KPNT=0
603 CONTINUE
    IF (KPRDRV.EQ.1) GO TO 604
    CALL PRNT(2)
604 CALL PROFIL (KPAR,XWS)
599 CONTINUE
    IF (MCO.NE.0) GO TO 30
    IF (KCMODE.EQ.0) GO TO 20
    DD=(X-XFIT)*SIG
    IF (DD.LT.0.) GO TO 20
    CHECK ATTEMPT TO REDUCE PROFILE THAT IS INFLECTED
    IF ((SIG*UINFL).GT.0.) GO TO 60
    REDUCED PROFILE FIT
    KCMODE=6, 6TH ORDER, KCMODE=4, 4TH ORDER
    KCMM=KCMODE/2-1
    GO TO (700,710), KCMM
    SWITCH TO 4TH ORDER
700 MODE=4
    CALL CFIT (IFIT,MODE)
    ILIM=3
    GO TO 720
    SWITCH TO 6TH ORDER
710 MODE=6
    CALL CFIT (IFIT,MODE)
    ILIM=4
720 CALL PROFIL (0,XWS)
    IS=IS-1
    XFITSV=XFIT
    KCMODE=0
    INPRO=4
    NT=MODE
    NTP=NT+1
    NTH=NT/2
    DXI=1.E-8*SIG
    PINL=P
    XI=X
    DO 730 I=1,ILIM
        J=IFIT(I)
        AUP(I,1)=ZW(J)
        AUP(I,2)=UW(J)
        AUP(I,3)=VW(J)
730 CONTINUE
    GO TO 390

32 IOUTR=1
30 CONTINUE

PRINT SUMMARY

C$$$$ CDC 6400 TIME RECORD
    ETIME=SECOND(A)
    DELT=ETIME-BTIME
C$$$$ CDC 6400 DATE RECORD

```

```

WHEN=DATE(0)
IF (SIG.GT.0.) PRINT 900
IF (SIG.LT.0.) PRINT 902
IF (IOUTR.EQ.1) PRINT 925,R
IF (IUNIFM.EQ.1) INPRO=2
IF (INPRO.EQ.2) PRINT 919
IF (INPRO.NE.2) PRINT 920
GO TO (930,935,940,943), INPRO
930 PRINT 932, NTSV
GO TO 945
935 PRINT 937, NTSV
IF (KCMODD.EQ.0) GO TO 945
PRINT 938, KCMODD,XFITSV
GO TO 945
940 PRINT 942, NTSV
GO TO 945
943 PRINT 939, NTSV
PRINT 950
PRINT 952, ((AUP(I,J), J=1,3), I=1,LIM)
PRINT 953
945 PRINT 921,U0,V0,RE
PRINT 922
DO 35 I=1,IS
PRINT 923, RSV(I),ETASV(I),PTSV(I),PSV(I),DPDRSV(I),TQUESV(I),
1 INUM(I),VMAG(I),VANG(I)
35 CONTINUE
IF (INFLT) 36,36,37
36 PRINT 960
GO TO 38
37 PRINT 980
DO 39 I=1,ICONT,2
PRINT 965, RINF(I)
J=I+1
IF (J.GT.ICONT) GO TO 39
PRINT 966,PINF(J)
39 CONTINUE
PRINT 980
38 CONTINUE
PRINT 970,DELT,WHEN
PRINT 975,KKONT
IF (MP.EQ.0.OR.MDI0G.EQ.0) CALL PRNT(3)
GO TO 1
40 IF(MP.EQ.0) GO TO 452
IF (MDI0G.NE.0) GO TO 45
452 IF (LC+NT-58) 450,450,451
451 CALL PRNT(4)
450 PRINT 200
CALL PRNT(2)
PRINT 202
LC=LC+4
45 CONTINUE
IF (KONT.GT.6) GO TO 55
DXI=DXI/2.0
GO TO 2
50 PRINT 203

```

```

      GO TO 1
51  PRINT 204
      GO TO 1
55  MDIOGT=MDIOG
      MDIOG=0
      CALL PRNT(3)
      MDIOG=MDIOGT
      GO TO 1
60  PRINT 985
      KCMODE=0
      GO TO 20
101 FORMAT (4I5,F15.0)
102 FORMAT (3F15.0)
103 FORMAT (2F15.0)
104 FORMAT (F15.0)
105 FORMAT (3F15.0)
200 FORMAT (35H INITIAL INCREMENT UNSATISFACTORY/10X15HERROR RANGE W
1AS)
202 FORMAT (2X21HHALVING THE INCREMENT/)
203 FORMAT (1H1,43HSINGULAR MATRIX ENCOUNTERED, JOB TERMINATED)
204 FORMAT (1H1,5X,54HMATCH-UP UNSUCCESSFUL IN 200 F-D STEPS, JOB TERM
1INATED)
900 FORMAT (1H1,5X,13HPUMP ANALYSIS/)
902 FORMAT (1H1,5X,16HTURBINE ANALYSIS/)
919 FORMAT(5X,8H SUMMARY,/5X,43H INCOMPRESSIBLE FLOW BETWEEN ROTATING
1DISKS,/5X,23H BOYACK INTEGRAL METHOD,29H MATCHED TO BOYD F-D START
1-UP/)
920 FORMAT(5X,8H SUMMARY,/5X,43H INCOMPRESSIBLE FLOW BETWEEN ROTATING
1DISKS,/5X,23H BOYACK INTEGRAL METHOD,/)
921 FORMAT(//5X,5H UO =,F8.4,5X,5H VO =,F8.4,5X,5H RE =,F8.4,/)
922 FORMAT(9X,1HR,6X,3HETA,10X,2HPT,12X,1HP,10X,4HDOPDR,11X,1HT,
1 18X,5HKKONT,9X,4HVMAG,9X,4HVVANG/)
923 FORMAT(6X,F5.3,1X,F8.4,1X,F12.6,1X,F12.6,1X,F12.6,1X,F12.6,15X,I8,
1 5X,F8.4,5X,F7.2)
925 FORMAT(//,10X,34HMINIMUM ALLOWABLE STEP SIZE AT R =,F10.5,/)
932 FORMAT (5X,I2,17HTH ORDER SOLUTION,5X,24HPARABOLIC INLET PROFILES/
1)
937 FORMAT (5X,I2,17HTH ORDER SOLUTION,5X,22HUNIFORM INLET PROFILES/)
938 FORMAT (/6X,21HPROFILE REDUCTION TO I1,15HTH ORDER AT R =F6.3/)
939 FORMAT (5X,I2,17HTH ORDER SOLUTION,5X,18HARBITRARY PROFILES/)
942 FORMAT (5X,I2,17HTH ORDER SOLUTION,5X,17HMATSCH ASYMPTOTIC/)
950 FORMAT (/ ,32X,1HZ,10X,1HU,10X,1HV/)
952 FORMAT (28X,F8.6,F11.6,F11.6)
953 FORMAT (1X,/)
960 FORMAT (//5X,23HNO U PROFILE INFLECTION/)
965 FORMAT (6X,24HU PROFILE INFLECTS AT R=F9.6)
966 FORMAT (6X,26HU PROFILE UNINFLECTS AT R=F9.6)
970 FORMAT (6X,19HTIME IN EXECUTION =F8.2,2X,7HSECONDS,5X,8HCDC 6400,
1 5X,A10/)
975 FORMAT (6X,19HINTEGRATION STEPS =I5)
980 FORMAT (//)
985 FORMAT (1H1,5X,66HPROFILE REDUCTION REQUIRED USING INFLECTED PROFI
1LE, NO REDUCTION )
7000 FORMAT (I2)
7005 FORMAT (4F15.0)

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7015 FORMAT (3I5,2F15.0)
END

```

SUBROUTINE PROFIL (KPAR,XWS)
COMMON QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
1      Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
2      Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
3      X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
4      M3,M4,MT,NT,RE,UO,VO,DPDR,P,P1,P2,P3,P4,PG1,PG2,
5      MODE,NOPNT,INPRO,KKONT,MDIOG,KF,KFF,KPNT,SIG,ALPRM,
6      ATES(64,65),XTES(64),
7      RSV(31),ETASV(31),PTSV(31),PSV(31),TQUESV(31),
8      DPDRSV(31),ACD(13),ZINF(13),ZVUV(21,13),ZVW(21,13),
9      INUM(31),RINF(11),AUP(7,3),UW(21),VW(21),RI1,
1     ZW(21),WW(21),Q(13,13),XPNT(31),RI2,IS,LU,NTP,PI,NTH,
2     VMAG(31),VANG(31)
EQUIVALENCE (X,R)
RI3=0.0
IF (SIG .GE.0.) RI2=0.0
DO 610 I=1,NTP,2
DO 610 J=1,NTP,2
RI3=RI3+AC(I)*BC(J)*F(I,J)
IF (SIG .LT.0.) GO TO 610
DO 605 K=1,NTP,2
IJK=I+J+K-2
RI2=RI2+(AC(I)*AC(J)*AC(K)+AC(I)*BC(J)*BC(K))/((IJK)*
1 2.0**IJK)
605 CONTINUE
610 CONTINUE
TORQ=-4.0*PI*(RI1-RI3*R*R)
PTOTL=RI2/UO
IF (SIG .GE.0.) PTOTL=PTOTL*R
PTOTL=PTOTL+P
ETA=-TORQ/(2.*PI*UO*PTOTL)
IF (SIG .GE.0.) ETA=-1./ETA
C STORE PERFORMANCE PARAMETERS
IF (KPAR.NE.1) GO TO 620
KPAR=J
IS=IS+1
C NEXT X PRINT STATION
ISS=IS+1
XWS=XPNT(ISS)
INUM(IS)=KKONT
PSV(IS)=P
DPDRSV(IS)=DPDR
RSV(IS)=R
ETASV(IS)=ETA
PTSV(IS)=PTOTL
TQUESV(IS)=TORQ
CALL VECTOR(VELMAG,VELANG)
VMAG(IS)=VELMAG
VANG(IS)=VELANG
620 CONTINUE
IF (NOPNT.EQ.1) RETURN
IF (SIG.GT.0.) PRINT 900
IF (SIG.LT.0.) PRINT 902
PRINT 901
GO TO (830,835,840,845), INPRO

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830 PRINT 932,NT
GO TO 850
835 PRINT 937,NT
GO TO 850
840 PRINT 942,NT
GO TO 850
845 PRINT 939,NT
850 CONTINUE
PRINT 909, UO,R,VO,DX,RE,KKONT
PRINT 910, DPDR,P,PTOTL,ETA,TORQ
PRINT 904
SUM=0.
DO 505 I=3,NTP,2
IARG=I-3
505 SUM=SUM+(FLOAT(I-1)*FLOAT(I-2)*AC(I))/(2**IARG)
SUM=SUM/RE+R
COMPT=ABS((SUM-DPDR)/DPDR)*100.
DO 506 J=1,NTH
K=2*J-1
ACD(K)=Z(J)
506 CONTINUE
IF (NT.EQ.4) ACD(5)=-16.*(Z(1)+Z(2)/4.)
IF (NT.EQ.6) ACD(7)=-64.*(Z(1)+Z(2)/4.+Z(3)/16.)
IF (NT.EQ.8) ACD(9)=-256.*(Z(1)+Z(2)/4.+Z(3)/16.+Z(4)/64.)
IF (NT.GT.8)ACD(NTP)=ALPRM
DO 510 I=1,NTP,2
DO 510 J=1,NTP,2
FJM=FLOAT(J-1)/FLOAT(I)
Q(I,J)=AC(I)*ACD(J)-FJM*AC(J)*(AC(I)/R+ACD(I))-BC(I)*BC(J)/R
510 CONTINUE
CSUM=0.0
DO 515 I=1,NTP,2
DO 515 J=1,NTP,2
CSUM=CSUM+FLOAT(I)*Q(I,J)/QS(I,J)
515 CONTINUE
CSUM=-12.*UO/(R*RE)-24.*CSUM
COMPTC=ABS((CSUM-DPDR)/DPDR)*100.
UW(1)=AC(1)
VW(1)=BC(1)
WW(1)=0.0
DO 521 I=2,LU
UW(I)=0.0
VW(I)=0.0
WW(I)=0.0
DO 518 J=1,NTP,2
UW(I)=UW(I)+AC(J)*ZVUV(I,J)
VW(I)=VW(I)+BC(J)*ZVUV(I,J)
FJ=J
WW(I)=WW(I)-(AC(J)/R+ACD(J))*ZVW(I,J)/FJ
518 CONTINUE
521 CONTINUE
DO 525 I=1,LU
PRINT 905, ZW(I),UW(I),VW(I),WW(I)
525 CONTINUE
PRINT 906

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DO 530 I=1,NTP,2
K=I-1
PRINT 907, K, AC(I), BC(I)
530 CONTINUE
PRINT 915
PRINT 911, COMPT, COMPTC
IF (MP.EQ.0.OR.MDI0G.EQ.0) CALL PRNT(4)
RETURN
900 FORMAT (1H1,5X,13HPUMP ANALYSIS/)
901 FORMAT(18X,32H REGULAR INTEGRAL SOLUTION ,//,14X,43HFULL ADMI
1SSION, INCOMPRESSIBLE LAMINAR FLOW)
902 FORMAT (1H1,5X,16HTURBINE ANALYSIS/)
904 FORMAT(13X,1HZ,11X,1HU,13X,1HV,13X,1HW,/)
905 FORMAT(10X,F6.3,4X,F10.6,4X,F10.6,4X,F10.6)
906 FORMAT(///,15X,35HU AND V POLYNOMIAL COEFFICIENTS ARE,//,16X,1HI,
19X,4HA(I),12X,4HB(I),/,15X,3H***,7X,6H*****,10X,6H*****,//)
907 FORMAT(15X,I2,4X,E12.5,5X,E12.5)
909 FORMAT(///,16X,5HU0 =,F7.3,11X,6HR =,E12.5,/16X,5HV0 =,F7.3,
111X,6HDR =,E12.5,/16X,5HRE =,F7.3,11X,6HSTEP =,I12)
910 FORMAT(//,26X,6HDPDR =,F10.5,/26X,6HP =,F10.5,/26X,6HPT =,
1F10.5,//26X,6HETA =F10.5,/26X,6HT =,F10.5,/)
911 FORMAT ( /,15X,12HC/L-T0-WALL=F8.2,2X,7HPC DIFF,/,15X,12HC/L-T0-FL
10W=F8.2,2X,7HPC DIFF)
915 FORMAT (/,15X,31HPRESSURE DERIVATIVE COMPARISONS/)
932 FORMAT (5X,I2,17H17TH ORDER SOLUTION,5X,24HPARABOLIC INLET PROFILES/
1)
937 FORMAT (5X,I2,17H17TH ORDER SOLUTION,5X,22HUNIFORM INLET PROFILES/)
942 FORMAT (5X,I2,17H17TH ORDER SOLUTION,5X,17HMATSCH ASYMPTOTIC/)
939 FORMAT (5X,I2,17H17TH ORDER SOLUTION,5X,18HARBITRARY PROFILES/)
END

```

SUBROUTINE INPUT(KER,AUP,R,IFIT ,RE,UO,VO,MODE,INPRO,SIG,P,LU,
1 KK1,KK2,KK3,PCTU,PCTV)

C THIS ROUTINE COMPUTES THE INPUT U,V VELOCITY PROFILES
C

COMMON FILL(889),A(64,65),X(64),DUM(212),ZVUV(21,13)
DIMENSION AUP(7,3),AC(13),BC(13),ZT(21),UT(21),VT(21)
DIMENSION Z2(7),Z4(7),Z6(7),Z8(7),Z10(7),Z12(7),Z14(7)
DIMENSION B(7,7),IFIT(7),VC(21)
EQUIVALENCE (FILL(677),AC(1)), (FILL(690),BC(1))
EQUIVALENCE (DUM(156),PDRIV)
NTP=MODE+1
LIM=MODE/2+1
LIMP1=LIM+1
GO TO (1,2,3,4), INPRO

C PARABOLIC VELOCITY PROFILES
C

1 UMAX=1.5*UO
VMAX=1.5*VO
AC(1)=UMAX
AC(3)=-4.*UMAX
BC(1)=VMAX
BC(3)=-4.*VMAX
300 RETURN

C UNIFORM PROFILES
C

2 CONTINUE
KER=0
K1=KK1
K2=KK2
50 CALL TESLA (UO,VO,RE,SIG,KER,ZT,UT,VT,P,UA,R,K1,K2,KK3,PDRIV)
IF (KER.GE.2) RETURN
UTEMP=UO
DO 100 I=1,LIM
J=IFIT(I)
AUP(I,1)=ZT(J)
AUP(I,2)=UT(J)
AUP(I,3)=VT(J)
100 CONTINUE
DO 110 I=1,LIM
A(I,LIMP1)=AUP(I,2)
Z7=AUP(I,1)
Z7SQ=Z7*Z7
B(I,1)=1.
A(I,1)=1.
DO 110 J=2,LIM
JM1=J-1
B(I,J)=B(I,JM1)*Z7SQ
A(I,J)=B(I,J)
110 CONTINUE
GO TO 400

C MATSCH 3RD APPROX ASYMPTOTIC PROFILES
C

3 DELTAZ=0.5/FLOAT(LIM-1)
DO 210 J=1,LIM

```

FJ=J
ZZ =DELTAZ*(FJ-1.)
RE2=RE*RE
RE3=RE*RE*RE
UR=U0/R
U2R=(U0*U0)/(R*R*R)
U3R=(U0*U0*U0)/(R*R*R*R*R)
U4R=(U0*U0*U0*U0)/(R*R*R*R*R*R*R)
Z2(J)=ZZ*Z7
Z4(J)=Z2(J)*Z2(J)
Z6(J)=Z4(J)*Z2(J)
Z8(J)=Z4(J)*Z4(J)
Z10(J)=Z6(J)*Z4(J)
Z12(J)=Z6(J)*Z6(J)
Z14(J)=Z8(J)*Z6(J)
T1=-12.*(Z2(J)*.5-.125)*UR
T2=-RE*U2R*(1.2*Z6(J)-1.5*Z4(J)+99.*Z2(J)/280.-3./224.)
T3=-RE2*UR*(-Z6(J)/15.+25*Z4(J)-39.*Z2(J)/560.+19./6720.)
T4=RE3*U2R*(-Z10(J)/90.+3.*Z8(J)/56.-23.*Z6(J)/240.+5.*Z4(J)/64.
1 -29793.*Z2(J)/1774080.+4361./7096320.)
T5=RE2*U3R*(-48.*Z10(J)/175.+9.*Z8(J)/14.-171.*Z6(J)/350.
1 +99.*Z4(J)/560.-2217.*Z2(J)/86240.+5301./6899200.)
T6=RE3*U4R*(-162.*Z14(J)/15925.+129.*Z12(J)/3850.-409.*Z10(J)/
1 9800.+893.*Z8(J)/39200.-879.*Z6(J)/156800.+99.*Z4(J)/125440.)
T7=RE3*U4R*(-876231.*Z2(J)/12556554000.+96151./5022617600.)
UASYM=T1+T2+T3+T4+T5+T6+T7
VASYM= R-(RE*UR*(Z4(J)-1.5*Z2(J)+(5./16.)))-(RE2*U2R*((3.*Z8(J)/
1 70.)-.1*Z6(J)+(33.*Z4(J)/560.)-(3.*Z2(J)/224.)+(19./17920.)))
2 +(RE3*U3R*((4.*Z12(J)/385.)-(4.*Z10(J)/105.)+(12.*Z8(J)/245.)-
3 (57.*Z6(J)/2100.)+(3.*Z4(J)/448.)-(3127./20697600.)))
A(J,LIMP1)=UASYM
AUP(J,3)=VASYM
DO 210 I=1,LIM
K=I*2-2
B(J,I)=ZZ**K
A(J,I)=B(J,I)
210 CONTINUE
GO TO 400
C
C ARBITRARY VELOCITY PROFILES OR AC(I) AND BC(I) ON AUP(I,J) DATA CARDS
4 IF (AUP(1,1).GT.0.5) GO TO 500
DO 310 I=1,LIM
ZZ=AUP(I,1)
A(I,LIMP1)=AUP(I,2)
ZZSQ=ZZ*ZZ
B(I,1)=1.
A(I,1)=1.
DO 310 J=2,LIM
JM1=J-1
B(I,J)=B(I,JM1)*ZZSQ
A(I,J)=B(I,J)
310 CONTINUE
C
400 CALL GAUSSN (A,X,64,65,LIM,KER)
IF (KER.EQ.2) RETURN

```

```

      DO 410 I=1,LIM
      K=I*2-1
      AC(K)=X(I)
410  CONTINUE
      DO 420 I=1,LIM
      A(I,LIMP1)=AUP(I,3)
      DO 420 J=1,LIM
      A(I,J)=B(I,J)
420  CONTINUE
      CALL GAUSSN (A,X,64,65,LIM,KER)
      IF (KER.EQ.2) RETURN
      DO 430 I=1,LIM
      K=I*2-1
      BC(K)=X(I)
430  CONTINUE
C     COMPUTE UO BY INTEGRATION OF U PROFILE
      UO=0.
      DO 440 I=1,LIM
      K=I*2-1
      FK=2**K
      FKK=FK*K
      UO=UO+AC(K)/FKK
440  CONTINUE
      UO=UO*2.*R
      IF(INPRO.EQ.2) GO TO 600
      RETURN

C
500  UO=0.
      DO 510 I=1,LIM
      K=2*I-1
      AC(K)=AUP(I,2)
      BC(K)=AUP(I,3)
      FK=2**K
      FKK=FK*K
      UO=UO+AC(K)/FKK
510  CONTINUE
      UO=UO*2.*R
      RETURN
600  CONTINUE
      DO 610 I=1,LU
      VC(I)=0.
      DO 610 J=1,NTP,2
      VC(I)=VC(I)+BC(J)*ZVUV(I,J)
610  CONTINUE
      MAR=1
      IF (LU.EQ.11) MAR=2
      J=0
      KV=0
      DO 620 I=1,19,MAR
      J=J+1
      VDUM=ABS((VC(J)-VT(I))/VT(I))*100.
      IF (VDUM.GT.PCTV) KV=KV+1
620  CONTINUE
      KU=0
      UDIF=ABS((UO-UA)/UO)*100.

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70

```
IF (UDIF.GT.PCTU) KU=1
IF (KK3.EQ.1) PRINT 630, (I,AC(I),I,BC(I), I=1,NTP,2)
630 FORMAT (5X,3HAC(,I2,2H)=E18.11,5X,3HBC(,I2,2H)=E18.11)
IF (KU.EQ.0.AND.KV.EQ.0) RETURN
K1=K1+10
KER=3
IF (K1 .GT.200) RETURN
K2=10
KER=-1
UO=UTEMP
GO TO 50
END
```

```

SUBROUTINE TESLA (U0,V0,RE,SIG,KER,Z,U,V,P,UA,R,KK1,KK2,KK3,PDRIV)
COMMON FILL(889),A(64,65),X(64)
DIMENSION U(21), V(21), W(21), DU(21), OV(21), DW(21)
DIMENSION S(21),Z(21)
KCMAX=KK1
IF (KK2.LE.0.OR.KK2.GT.KK1) KK2=KK1
KCCMX=KK2
KPRNT=KK3
IF (KER.EQ.-1) GO TO 350
FJW=21.
JW=21
DZ = .5/(FJW-1.)
JWM4 = JW-4
JWM3 = JW-3
JWM2 = JW-2
JWM1 = JW-1
JWP1 = JW+1
JWP2 = JW+2
JWP3 = JW+3
JW2M6 = 2*JW-6
JW2M5 = 2*JW-5
JW2M4 = 2*JW-4
JW2M3 = 2*JW-3
JW2   = 2*JW
JW2P1 = 2*JW+1
JW2P2 = 2*JW+2
JW2P3 = 2*JW+3
JW3M6 = 3*JW-6
JW3M5 = 3*JW-5
JW3M4 = 3*JW-4
JW3M3 = 3*JW-3
JW3M2 = 3*JW-2
JW3M1 = 3*JW-1
JW3   = 3*JW
JW3P1 = 3*JW+1
JW3P2 = 3*JW+2
JWM1D2=JWM1/2
R = 1.
C UNIFORM VELOCITY PROFILES AT R=1
DO 65 J=1,JW
U(J) = U0
V(J) = V0
W(J) = 0.
65 CONTINUE
U(JW) = 0.
V(JW) = R
C INTEGRAL (USQ+VSQ) AT R=1, Z=0 TO Z=.5
51 DO 52 J=1, JW
S(J) = (U(J)*U(J)) + (V(J)*V(J))
52 CONTINUE
XP = 0.
DO 56 J=1, JWM1D2
JJ = 2*J
JJJ = JJ+1
XPT = (4.*S(JJ)) + (2.*S(JJJ))

```

```

XP = XP+XPT
56 CONTINUE
XP = (XP+S(1)-S(JW)) * DZ / 3.
TERM1 = 2. * XP
TERM2 = -2. * (U(JWM2) - 4. * U(JWM1) + 3. * U(JW))
DPDR = TERM1 + TERM2
PDRIV=DPDR
200 CONTINUE
ETA = 0.
T = 0.
205 DO 210 J=1,JW
FJ = J
Z(J) = (FJ-1.) * DZ
210 CONTINUE
C TORQUE, UV PRODUCT, R=1
C INTEGRAL UV AT R=1, Z=0 TO Z=.5
250 XO = 0.
DO 255 J=1, JWM1D2
JJ = 2*J
JJJ = JJ+1
XOT = (4.*U(JJ)*V(JJ)+(2.*U(JJJ)*V(JJJ)))
XO = XO + XOT
255 CONTINUE
XO = (XO+(U(1)*V(1)) - (U(JW)*V(JW))) * DZ/3.
C DYNAMIC PRESSURE, R=1
C INTEGRAL (USQ+VSQ)*U AT R=1, Z=0 TO Z=.5
DO 260 J=1, JW
S(J) = ((U(J)*U(J))+(V(J)*V(J)))*U(J)
260 CONTINUE
XP = 0.
DO 265 J=1,JWM1D2
JJ = 2*J
JJJ = JJ+1
XPT = (4.*S(JJ))+(2.*S(JJJ))
XP = XP+XPT
265 CONTINUE
XP = (XP+S(1) - S(JW))*DZ/3.
PT = P + (XP/U0)
B1 = RE*DZ*DZ
BD1=1./B1
BD2=2./B1
B2 = 2.*DZ
BD2=1./B2
DR=1.E-6*SIG
KCC=0
KC = 0
GO TO 1000
350 R = R + DR
RDR=1./DR+1./R
DPDRP = DPDR
DO 365 M=1, JW3P1
DO 360 N=1, JW3P2
A(M,N) = 0.
360 CONTINUE
365 CONTINUE

```

```

DO 370 M = 1, JWM2
A(M,M+1) = (U(M+1)/DR) + B02
A(M,M) = -(W(M+1)/B2) - B01
A(M,M+2) = (W(M+1)/B2) - B01
MPJWP1=M+JWP1
A(M,MPJWP1) = -2. * V(M+1) / R
MJW2P1=M+JW2P1
A(M,MJW2P1) = (U(M+2)/B2) - (U(M)/B2)
A(M,JW3P1) = 1.
A(M,JW3P2) = (V(M+1)*V(M+1)/R)-(W(M+1)*U(M+2)/B2)+(W(M+1)*U(M)/B2)
1 +((U(M+2)+U(M))/B1)-((2.*U(M+1))/B1)
370 CONTINUE
DO 380 M = JWM1, JW2M4
MMJWM3=M-JWM3
A(M,MMJWM3) = V(MMJWM3)/R
A(M,M+3) = (U(MMJWM3)/DR) + (U(MMJWM3)/R) + B02
A(M,M+2) = -(W(MMJWM3)/B2) - B01
A(M,M+4) = (W(MMJWM3)/B2) - B01
MMJWM4=M-JWM4
MMJWM2=M-JWM2
MPJWP3=M+JWP3
A(M,MPJWP3) = (V(MMJWM4) - V(MMJWM2))/B2
A(M,JW3P2) = (-U(MMJWM3)*V(MMJWM3))/R)-(W(MMJWM3)*(V(MMJWM4)
1 -V(MMJWM2))/B2)+(V(MMJWM4)/B1)-(2.*V(MMJWM3)/B1)+(V(MMJWM2)/B1)
380 CONTINUE
DO 390 M=JW2M3, JW3M6
MJW2M5=M-JW2M5
A(M,MJW2M5) = RDR
A(M,M+6)=B2D
A(M,M+4) =-B2D
MJW2M6=M-JW2M6
MJW2M4=M-JW2M4
A(M,JW3P2) = (-U(MJW2M5)/R)-((W(MJW2M6)-W(MJW2M4))/B2)
390 CONTINUE
A(JW3M5,JW) = 1.
A(JW3M4,JW2) = 1.
A(JW3M4,JW3P2) = R-V(JW)
A(JW3M3,JW3) = 1.
A(JW3M2,1) = -3.
A(JW3M2,2) = 4.
A(JW3M2,3) = -1.
A(JW3M2,JW3P2) = 3.*U(1)-4.*U(2)+U(3)
A(JW3M1,JWP1) = -3
A(JW3M1,JWP2) = 4.
A(JW3M1,JWP3) = -1.
A(JW3M1,JW3P2) = 3.*V(1)-4.*V(2)+V(3)
A(JW3,JW2P1) = 1.
A(JW3,JW3P2) = -W(1)
A(JW3P1,JW3M2) = 1.
A(JW3P1,JW3M1) = -4.
A(JW3P1,JW3P2) = 4.*W(JWM1)-W(JWM2)
CALL GAUSSN (A, X, JW3P1, JW3P2, JW3P1, KER)
IF (KER.EQ.2) RETURN
395 DO 400 J=1,JW
DU(J) = X(J)

```

```

JPJW=J+JW
JPJW2=J+JW2
DV(J) = X(JPJW)
DW(J) = X(JPJW2)
DPDR = X (JW3P1)
400 CONTINUE
DO 410 J=1,JW
U(J) = U(J) + DU(J)
V(J) = V(J) + DV(J)
W(J) = W(J) + DW(J)
410 CONTINUE
U(JW) = 0.
V(JW) = R
W(JW) = 0.
C CALCULATE PRESSURE
500 P = P + (DPDRP+DPDR) * .5 * DR*SIG
KCC=KCC+1
KC=KC+1
IF (KC.GT.101) GO TO 520
FKC=KC
DR=(1.E-6+9.99E-8*FKC*FKC)*SIG
GO TO 530
520 DR=.001*SIG
530 CONTINUE
IF (KC.GE.KCMAX) GO TO 600
599 IF (KCC.NE.KCCMX) GO TO 350
KCC=0
600 XI = 0.
IF (SIG.GT.0.) XP=0.
DO 610 J=1, JWM1D2
JJ = 2*J
JJJ = JJ+1
C TORQUE, R
XIT = (4.*U(JJ)*V(JJ)) + (2.*U(JJJ)*V(JJJ))
XI = XI+XIT
IF (SIG.LT.0.) GO TO 610
C DYNAMIC PRESSURE, R
XPT=4.*(U(JJ)*U(JJ)*U(JJ)+V(JJ)*V(JJ)*U(JJ))+2.*(U(JJJ)*U(JJJ)*
1 U(JJJ)+V(JJJ)*V(JJJ)*U(JJJ))
XP=XP+XPT
610 CONTINUE
XI = (XI+(U(1)*V(1)) - (U(JW)*V(JW))) * DZ/3.
IF (SIG.LT.0.) GO TO 611
XP=(XP+ U(1)*U(1)*U(1)+V(1)*V(1)*U(1)-U(JW)*U(JW)*U(JW)-
1 V(JW)*V(JW)*U(JW))*DZ/3.
611 CONTINUE
PT=P+XP*R/U0
IF (SIG.LT.0.) PT=P+XP/U0
RSQ = R*R
T = -(X0-(RSQ*XI)) * 12.566370606
WI = 6.283185307 * PT * U0
ETA=WI/T
IF (SIG.GT.0..AND.R.EQ.1.) ETA=0.
IF (SIG.LT.0.) ETA=-T/WI
IF (KPRNT.EQ.1) GO TO 1001

```

```

      GO TO 1085
1000 IF (KPRNT.EQ.0) GO TO 350
1001 PRINT 1
      1 FORMAT (1H1)
      PRINT 1005
1005 FORMAT (50H FLOW BETWEEN ROTATING DISKS, INCOMPRESSIBLE FLUID,/)
      PRINT 1010
1010 FORMAT (34H BOYD ANALYSIS, PROGRAMMED BY RICE,/)
1020 PRINT 1021
1021 FORMAT (26H UNIFORM VELOCITY PROFILES,/)
1040 PRINT 1041, UO, VO, RE
1041 FORMAT (1X, 4HUO =F8.4,5X,4HVO =F8.4,5X, 4HRE =F8.4,/)
      PRINT 1043, R , DR ,KC
1043 FORMAT (4H R =F12.8,10X,4HDR =F12.8,9X,3HKC=I5/)
      PRINT 1045, DPDR, P, PT
1045 FORMAT (7H DPDR =F15.6,5X,3HP =F15.8,5X,4HPT =F15.8,/)
      PRINT 1050, T, ETA
1050 FORMAT (4H T =F15.8,10X,5HETA =F10.5,/)
      PRINT 1055
1055 FORMAT (3X,2H Z,14X,2H U,17X,2H V,17X,1HW,/)
      DO 1070 J=1,JW
      PRINT 1060, Z(J), U(J), V(J), W(J)
1060 FORMAT (F6.3,3(4X,F15.8))
1070 CONTINUE
1085 CONTINUE
C COMPUTE CONTINUITY
  UA=0.
  DO 1300 J=1,10
    JJ=J*2
    JJJ=JJ+1
    UAA=4.*U(JJ)+2.*U(JJJ)
    UA=UA+UAA
1300 CONTINUE
    UA=(UA+U(1)-U(21))*2.*R*0.025/3.
    COMPR=((UA-UO)/UO)*100.
    IF (KPRNT.EQ.0) GO TO 1090
    PRINT 1080 ,UA,COMPR
1080 FORMAT (6X,15HF-D CONTINUITY=F10.6,5X,5HDIFF=F6.3,3H PC)
1090 IF (KC.NE.KCMAX) GO TO 350
      RETURN
      END

```

```
      SUBROUTINE CFIT (IFIT,MODE)
C      CURVE FIT POINTS CORRESPONDING TO ORDER OF FIT
      DIMENSION IFIT(7)
C      NOTE THIS SUB MUST HAVE 11 OR 21 PT FIT ADDED
      KDIR=MODE/2-1
      GO TO(4,6,8,10,12), KDIR
C      4TH ORDER INTEGRAL REDUCTION (11 PT)
      4 IFIT(1)=1
        IFIT(2)=7
        IFIT(3)=11
        RETURN
C      6TH ORDER INTEGRAL REDUCTION (11 PT)
      6 IFIT(1)=1
        IFIT(2)=5
        IFIT(3)=8
        IFIT(4)=11
        RETURN
C      8TH ORDER F-D MATCH-UP (21 PT)
      8 IFIT(1)=1
        IFIT(2)=6
        IFIT(3)=11
        IFIT(4)=16
        IFIT(5)=21
        RETURN
      10 CONTINUE
        RETURN
      12 CONTINUE
        RETURN
      END
```

C
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C

SUBROUTINE GAUSSN (A,X,NN,MM,N,KER)

DIMENSION A(NN,MM),X(NN)

A IS AN N BY N+1 MATRIX OF COEFFICIENTS FOR A SET OF N
SIMULTANEOUS LINEAR ALGEBRAIC EQUATIONS. THE CONSTANTS ARE
STORED IN COLUMN N+1. X IS THE N DIMENSIONAL ANSWER VECTOR

THIS SUBROUTINE HAS VARIABLE DIMENSIONS (NN AND MM)

A IS DIMENSIONED A(NN,MM) AND X IS DIMENSIONED X(NN)

NPM=N+1

EP=1.E-27

```
10 DO 34 L=1,N
    KP=0
    Z=0.0
    DO 12 K=L,N
    IF(Z-ABS(A(K,L)))11,12,12
11 Z=ABS(A(K,L))
    KP=K
12 CONTINUE
    IF(L-KP)13,20,20
13 DO 14 J=L,NPM
    Z=A(L,J)
    A(L,J)=A(KP,J)
14 A(KP,J)=Z
20 IF(ABS(A(L,L))-EP)50,50,30
30 IF(L-N)31,40,40
31 LP1=L+1
    DO 34 K=LP1,N
    IF(A(K,L))32,34,32
32 RATIO=A(K,L)/A(L,L)
    DO 33 J=LP1,NPM
33 A(K,J)=A(K,J)-RATIO*A(L,J)
34 CONTINUE
40 DO 43 I=1,N
    II=N+1-I
    S=0.0
    IF(II-N)41,43,43
41 IIP1=II+1
    DO 42 K=IIP1,N
42 S=S+A(II,K)*X(K)
43 X(II)=(A(II,NPM)-S)/A(II,II)
    KER=1
    GO TO 75
50 KER=2
75 CONTINUE
RETURN
END
```

SUBROUTINE ITRO (K)

THIS ROUTINE CHECKS THE CURRENT VALUE OF X WITH THE SPECIFIED
CUT-OFF VALUE OF X

C
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C

```

COMMON QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
1      Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
2      Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
3      X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
4      M3,M4,MT,NT,RE,UO,VO,DPDR,P,P1,P2,P3,P4,PG1,PG2,
5      MODE,NOPNT,INPRO,KKONT,MOIOG,KF,KFF,KPNT,SIG,ALPRM
EQUIVALENCE (X,R)
IF (SIG *(X+DX-XC)) 2,1,1
1  IF (M4) 4,5,4
2  K=0
3  RETURN
5  DX=XC-X
   KONT =1
   M4=1
   GO TO 3
4  K=1
   GO TO 3
END

```

SUBROUTINE MOVE

THIS ROUTINE RE-STORES THE VARIABLES OF PREVIOUS STEPS BEFORE
PROCEEDING TO CALCULATIONS FOR THE NEXT STEP

C
C
C
COMMON QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
1 Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
2 Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
3 X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
4 M3,M4,MT,NT,RE,UO,VO,DPDR,P,P1,P2,P3,P4,PG1,PG2,
5 MODE,NOPNT,INPRO,KKONT,MDIOG,KF,KFF,KPNT,SIG,ALPRM

EQUIVALENCE (X,R)

P4=P3

P3=P2

P2=P1

P1=P

PG2=PG1

PG1=DPDR

X4=X3

X3=X2

X2=X1

X1=X

DO 1 I=1,NT

Y4(I)=Y3(I)

Y3(I)=Y2(I)

Y2(I)=Y1(I)

Y1(I)=Y(I)

Z4(I)=Z3(I)

Z3(I)=Z2(I)

Z2(I)=Z1(I)

Z1(I)=Z(I)

1 CONTINUE

RETURN

END

SUBROUTINE PRNT (K)

THIS ROUTINE PROVIDES PRINTED OUTPUT FOR EACH INTEGRATION STEP WHEN THE INPUT VARIABLE MP=0. THIS PRINTOUT MAY BE CONSIDERED LARGELY DIAGNOSTIC IN NATURE AS IT DOES NOT INCLUDE U, V, W OR TURBINE PERFORMANCE INFORMATION

```

COMMON QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
1      Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
2      Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
3      X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
4      M3,M4,MT,NT,RE,UO,VO,DPDR,P,P1,P2,P3,P4,PG1,PG2,
5      MODE,NOPNT,INPRO,KKONT,MDIOG,KF,KFF,KPNT,SIG,ALPRM

```

EQUIVALENCE (X,R)

LF=1

IF (MT.NE.0) LF=2

IF(K-3) 50,3,80

50 IF(K-1) 2,1,2

K=1 PRINT INITIAL VALUES, HEADINGS

1 IF(MT) 10,10,11

10 PRINT 200

PRINT 206

12 PRINT 201,XI,DXI,DXM,XC,EMIN,EMAX

DO 6 I=1,NT

PRINT 208, I,Y(I),I,Z(I)

6 CONTINUE

PRINT 209

LC=11+NT

GO TO 60

11 PRINT 202

PRINT 206

PRINT 206

GO TO 12

K=2 PRINT INTEGRATION STEP

2 IF(LC+NT-58) 20,20,70

70 GO TO (71,72),LF

71 PRINT 200

PRINT 209

LC=6

GO TO 20

72 PRINT 202

LC=6

PRINT 209

20 PRINT 203, X,DX,P,DPDR,KKONT

DO 4 I=1,NT

PRINT 205,I,Y(I),I,Z(I),I,E(I)

4 CONTINUE

PRINT 206

LC=LC+2+NT

GO TO 60

K=3 PRINT FINAL CUTOFF INTEGRATION STEP

3 IF(LC+NT-58)30,30,73

73 GO TO (74,75),LF

74 PRINT 200

GO TO 30

```

75 PRINT 202
80 PRINT 211
   IF (MDI0G.NE.0) GO TO 59
   PRINT 204,XC
   DO 5 I=1,NT
   PRINT 210,I,Y(I),I,Z(I)
5   CONTINUE
   PRINT 207
59  CONTINUE
60  RETURN
   K=4   INDEX PAGE, PRINT HEADINGS
80  GO TO (81,82),LF
81  PRINT 200
   PRINT 209
   LC=6
   GO TO 60
82  PRINT 202
   PRINT 209
   LC=6
   GO TO 60
200 FORMAT (56H1   RUNGE-KUTTA-GILL / ADAMS-MOULTON INTEGRATION ROUTIN
1E)
201 FORMAT (20H  INITIAL CONDITIONS/16X1HX,17X5HDEL X,12X9HMAX DEL X,
113X,5HCUT X,12X,9HMIN ERROR,10X,9HMAX ERROR/5X,6E19.7)
202 FORMAT (40H1   RUNGE-KUTTA-GILL INTEGRATION ROUTINE)
203 FORMAT (2E18.10,10X,E15.7,5X,E15.7,10X,6HKKONT=I5)
204 FORMAT (/34H   FINAL RESULT AT A CUT-OFF X OF,E15.7)
205 FORMAT (10X,2HY(,I2,4H) = ,E15.7,4X5HDERV(,I2,4H) = ,E15.7,4X,
16HERROR(,I2,4H) = ,E15.7)
206 FORMAT (1H )
207 FORMAT (1H1/1H1)
208 FORMAT (9X,3HYI(,I2,4H) = ,E15.7,3X,6HDERVI(,I2,4H) = ,E15.7)
209 FORMAT (/12X19HINTEGRATION RESULTS/11X,1HX,13X,5HDEL X,23X,1HP,
1 19X,4HDPDR/20X,4HY(I),
223X,7HDERV(I),23X,8HERROR(I)/)
210 FORMAT (10X,2HY(,I2,4H) = ,E15.7,4X5HDERV(,I2,4H) = ,E15.7)
211 FORMAT(1H1)
   END

```

SUBROUTINE RKG1

THIS ROUTINE USES THE RUNGE-KUTTA-GILL (R-K-G) METHOD OF
INTEGRATION TO DETERMINE THE STARTING POINTS REQUIRED FOR
USE WITH THE A-M METHOD

```

COMMON QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
1     Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
2     Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
3     X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
4     M3,M4,MT,NT,RE,UO,VO,DPDR,P,P1,P2,P3,P4,PG1,PG2,
5     MODE,NOPNT,INPRO,KKONT,MDIOG,KF,KFF,KPNT,SIG,ALPRM
EQUIVALENCE (X,R)
DIMENSION AA(24)
D1=SQRT (0.5000)
D2=SQRT (2.0000)
D3=3.0*D1
C1=1.0-D1
C2=2.0-D2
C3=2.0-D3
C4=1.0+D1
C5=2.0+D2
C6=2.0+D3
DO 10 I=1,NT
AA(I)=Z(I)
10 CONTINUE
DO 1 I=1,NT
Y(I)=Y(I)+DX*Z(I)/2.0
1 CONTINUE
X=X1+DX/2.0
CALL DERV
DO 2 I=1,NT
Y(I)=Y(I)+C1*DX*(Z(I)-AA(I))
AA(I)=C2*Z(I)-C3*AA(I)
2 CONTINUE
CALL DERV
DO 3 I=1,NT
Y(I)=Y(I)+C4*DX*(Z(I)-AA(I))
AA(I)=C5*Z(I)-C6*AA(I)
3 CONTINUE
X=X1+DX
CALL DERV
DO 4 I=1,NT
Y(I)=Y(I)+DX*Z(I)/6.0-DX*AA(I)/3.0
4 CONTINUE
CALL DERV
P=P1+SIG *DPDR*DX
RETURN
END

```

SUBROUTINE ADMI

THIS ROUTINE UTILIZES THE ADAMS-MOULTON (A-M) PREDICTOR-CORRECTOR METHOD OF INTEGRATION OVER A FIXED INCREMENT OF THE INDEPENDENT VARIABLE IN ORDER TO EVALUATE THE DEPENDENT VARIABLES Y(I) AT THE VALUE OF THE INDEPENDENT VARIABLE X+DX

COMMON QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
 1 Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
 2 Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
 3 X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
 4 M3,M4,MT,NT,RE,UO,VO,DPDR,P,P1,P2,P3,P4,PG1,PG2,
 5 MODE,NOPNT,INPRO,KKONT,MDIOG,KF,KFF,KPNT,SIG,ALPRM

EQUIVALENCE (X,R)

DIMENSION YY(24)

IF(KF) 50,51,50

50 IF(KFF)52,53,52

52 KF=0

KFF=0

GO TO 51

53 KFF=1

51 CONTINUE

OBTAIN TENTATIVE VALUE OF NEXT POINT VIA ADAMS-MOULTON INTEGRATION

CHECK ERROR FOR POSSIBLE CHANGE IN STEP-SIZE

X=X1+DX

MM=0

D1=DX/24.0

D2=19.0/270.0

DO 5 I=1,NT

Y(I)=Y1(I)+D1*(55.0*Z1(I)-59.0*Z2(I)+37.0*Z3(I)-9.0*Z4(I))

YY(I)=Y(I)

5 CONTINUE

CALL DERV

DO 6 I=1,NT

Y(I)=Y1(I)+D1*(9.0*Z(I)+19.0*Z1(I)-5.0*Z2(I)+Z3(I))

6 CONTINUE

CALL DERV

C PRESSURE CALC FOR EACH REGULAR A-M STEP

QQ=X2*(X1*X1-X*X)+X1*(X*X-X2*X2)+X*(X2*X2-X1*X1)

IF(QQ.EQ. 0.0) QQ=1.0E-16

S1=(PG2*(X-X1)+PG1*(X2-X)+DPDR*(X1-X2))/QQ

S2=(DPDR-PG1)/(X-X1)-S1*(X+X1)

S3=PG2-S1*X2*X2-S2*X2

P=P1+SIG *(S3*(X-X1)+S2/2.*(X*X-X1*X1)+S1/3.*(X*X*X-X1*X1*X1))

DO 7 I=1,NT

E(I)=ABS (D2*(Y(I)-YY(I)))

IF(EMAX-E(I)) 45,1,1

1 IF(E(I)-EMIN) 8,7,7

45 MM=MM+1

GO TO 7

8 M3=M3+1

7 CONTINUE

IF(MM) 46,46,3

46 IF(M3-NT) 10,2,2

C DOUBLE INTEGRATION STEP

```

2   IF (MC) 10,21,10
21  IF (SIG *(X+4.*DX-XC)) 22,22,10
22  IF (SIG *(DX+DX-DXM)) 23,23,10
23  CONTINUE
    IF(LC+NT-58) 230,230,231
231 CALL PRNT(4)
230 IF (MP.EQ.0) GO TO 233
    IF (MDI0G.NE.0) GO TO 232
233 PRINT 201
    CALL PRNT(2)
    PRINT 204
    LC=LC+2
232 CONTINUE
    DX=DX+DX
    X1=X
    P1=P
    PG1=DPDR
    DO 25 I=1,NT
    Z3(I)=Z4(I)
    Z1(I)=Z(I)
    Y1(I)=Y(I)
25  CONTINUE
    KKONT=KKONT+1
    CALL RKGI
    DO 26 I=1,NT
    E(I)=0.0
26  CONTINUE
    IF (MDI0G.EQ.0) CALL PRNT(2)
    M1=1
    GO TO 13
C   HALVE INTEGRATION STEP
3   IF (M2) 30,40,30
30  CONTINUE
    M2=1
31  IF(LC+NT-58) 350,350,351
351 CALL PRNT(4)
350 IF (MP.EQ.0) GO TO 353
    IF (MDI0G.NE.0) GO TO 352
353 KFFP1=KFF+1
    GO TO (451,452), KFFP1
451 PRINT 202
    GO TO 453
452 PRINT 206
453 CONTINUE
    CALL PRNT(2)
    PRINT 204
    LC=LC+2
352 CONTINUE
    IF (M1) 34,32,34
32  IF (SIG *(X+DX+DX-XC)) 33,33,10
33  IF (KFF) 36,35,36
35  DX=DX/2.0
    DO 42 I=1,NT
    Y(I)=Y1(I)
    Z(I)=Z1(I)

```

```

42 CONTINUE
   KKONT=KKONT+1
   CALL RKG1
   DO 38 I=1,NT
   E(I)=0.0
38 CONTINUE
   IF (MDIOG.EQ.0) CALL PRNT(2)
   KONT=2
   KF=1
   KFF=0
   GO TO 13
34 X1=X2
   P1=P2
   M1=0
   DO 39 I=1,NT
   Y1(I)=Y2(I)
   Z1(I)=Z2(I)
39 CONTINUE
   GO TO 35
10 M2=1
   IF (M1) 11,13,11
11 M1=0
13 M3=0
   RETURN
36 KFF=0
   X1=X4
   P1=P4
   DO 41 I=1,NT
   Y(I)=Y4(I)
   Y1(I)=Y4(I)
   Z(I)=Z4(I)
   Z1(I)=Z4(I)
41 CONTINUE
   GO TO 35
40 Z2(1)=1.E12
   GO TO 13
201 FORMAT (51H      DOUBLING THE INTERVAL,      ERROR ARRAY AS FOLLOWS)
202 FORMAT (50H      HALVING THE INTERVAL,      ERROR ARRAY AS FOLLOWS)
203 FORMAT (8X,6HERROR(,I2,5H) = ,E15.9)
204  FORMAT (1H )
205  FORMAT(1H1)
206 FORMAT (53H      RE-HALVING THE INTERVAL,      ERROR ARRAY AS FOLLOWS)
   END

```

SUBROUTINE DERV

THIS ROUTINE EVALUATES THE DERIVATIVE VALUES AT EACH INCREMENT OF THE INDEPENDENT VARIABLE

COMMON QS(13,13),F(13,13),G(13,13),H(13,13),AC(13),BC(13),
 1 Y(12),Y1(12),Y2(12),Y3(12),Y4(12),Z(12),Z1(12),
 2 Z2(12),Z3(12),Z4(12),YI(12),E(12),XI,XC,X,X1,X2,
 3 X3,X4,DXI,DX,DXM,EMIN,EMAX,KONT,LC,MP,MC,M1,M2,
 4 M3,M4,MT,NT,RE,UO,VO,DPDR,P,P1,P2,P3,P4,PG1,PG2,
 5 MODE,NOPNT,INPRO,KKONT,MDIOG,KF,KFF,KPNT,SIG

EQUIVALENCE (X,R)

T11=1.0/2.0

T12=1.0/(3.0*8.0)

T13=1.0/(5.0*32.0)

T14=1.0/(7.0*128.0)

T15=1.0/(9.0*512.0)

BETA1=-UO/(2.0*R*R)

MODD=MODE/2-1

GO TO (100,200,300), MODD

THIS SECTION IS FOR FORTH ORDER POLYNOMIALS FOR U AND V

100 CONTINUE

IF (KONT.EQ.1) GO TO 120

AC(1)=Y(1)

AC(3)=Y(2)

AC(5)=-16.0*Y(1)-4.0*Y(2)

BC(1)=Y(3)

BC(3)=Y(4)

BC(5)=16.0*(R-Y(3))-4.0*Y(4)

120 CONTINUE

BETA3=0.0

BETA4=0.0

DO 140 I=1,5,2

DO 140 J=1,5,2

BETA3=BETA3+H(I,J)*BC(I)*BC(J)-G(I,J)*AC(I)*AC(J)

BETA4=BETA4+F(I,J)*AC(I)*BC(J)

140 CONTINUE

BETA2=-Y(1)*Y(3)/R+2.0*Y(4)/RE

BETA3=BETA3/R+6.0*UO/(R*RE)+(1.0*Y(2)+AC(5)/2.0)/RE

BETA4=-2.0/R*BETA4+(1.0*Y(4)+BC(5)/2.0)/RE

T24=Y(1)

T31=Y(1)*(G(1,1)+H(1,1))+Y(2)*(G(1,3)+H(3,1))+AC(5)*(G(1,5)+H(5,1)

1)

T32=Y(1)*(G(3,1)+H(1,3))+Y(2)*(G(3,3)+H(3,3))+AC(5)*(G(3,5)+H(5,3)

1)

T33=Y(1)*(G(5,1)+H(1,5))+Y(2)*(G(5,3)+H(3,5))+AC(5)*(G(5,5)+H(5,5)

1)

T41=F(1,1)*Y(3)+F(1,3)*Y(4)+F(1,5)*BC(5)

T42=F(3,1)*Y(3)+F(3,3)*Y(4)+F(3,5)*BC(5)

T43=F(5,1)*Y(3)+F(5,3)*Y(4)+F(5,5)*BC(5)

T44=F(1,1)*Y(1)+F(3,1)*Y(2)+F(5,1)*AC(5)

T45=F(1,3)*Y(1)+F(3,3)*Y(2)+F(5,3)*AC(5)

T46=F(1,5)*Y(1)+F(3,5)*Y(2)+F(5,5)*AC(5)

```

Z(1)=(BETA1*(T32-4.0*T33)-BETA3*(T12-4.0*T13))/((T11-16.0*T13)
1*(T32-4.0*T33)-(T12-4.0*T13)*(T31-16.0*T33))
Z(2)=(BETA1-(T11-16.0*T13)*Z(1))/(T12-4.0*T13)
Z(3)=BETA2/T24
Z(4)=((BETA4-16.0*T46)-(T41-16.0*T43)*Z(1)-(T42-4.0*T43)*Z(2)
1-(T44-16.0*T46)*Z(3))/(T45-4.0*T46)
DPDR=Y(3)*2/R+2.0*Y(2)/RE-Y(1)*Z(1)
RETURN

```

C
C

THIS SECTION IS FOR SIXTH ORDER POLYNOMIALS FOR U AND V

```

200 CONTINUE
IF (KONT.EQ.1) GO TO 220
AC(1)=Y(1)
AC(3)=Y(2)
AC(5)=Y(3)
AC(7)=-64.0*Y(1)-16.0*Y(2)-4.0*Y(3)
BC(1)=Y(4)
BC(3)=Y(5)
BC(5)=Y(6)
BC(7)=64.0*(R-Y(4))-16.0*Y(5)-4.0*Y(6)
220 CONTINUE
BETA5=0.0
BETA6=0.0
DO 240 I=1,7,2
DO 240 J=1,7,2
BETA5=BETA5+H(I,J)*BC(I)*BC(J)-G(I,J)*AC(I)*AC(J)
BETA6=BETA6+F(I,J)*AC(I)*BC(J)
240 CONTINUE
BETA5=BETA5/R+6.0*UO/(R*RE)+(AC(3)+AC(5)/2.0+3.0*AC(7)/16.0)/RE
BETA6=-2.0*BETA6/R+(BC(3)+BC(5)/2.0+3.0*BC(7)/16.0)/RE
BETA2=(-Y(1)*Y(4)/R+2.0*Y(5)/RE)/Y(1)
BETA3=12.0*Y(3)/RE+2.0/R*(Y(1)*Y(2)+Y(4)*Y(5))
BETA4=12.0*Y(6)/RE+(Y(1)*Y(5)-Y(2)*Y(4))/R
T25=1.0
T31=-Y(2)
T32=Y(1)
T41=-2.0*Y(5)
T45=Y(2)
T46=Y(1)
T51=Y(1)*(G(1,1)+H(1,1))+Y(2)*(G(1,3)+H(3,1))+Y(3)*(G(1,5)+H(5,1))
1+AC(7)*(G(1,7)+H(7,1))
T52=Y(1)*(G(3,1)+H(1,3))+Y(2)*(G(3,3)+H(3,3))+Y(3)*(G(3,5)+H(5,3))
1+AC(7)*(G(3,7)+H(7,3))
T53=Y(1)*(G(5,1)+H(1,5))+Y(2)*(G(5,3)+H(3,5))+Y(3)*(G(5,5)+H(5,5))
1+AC(7)*(G(5,7)+H(7,5))
T54=Y(1)*(G(7,1)+H(1,7))+Y(2)*(G(7,3)+H(3,7))+Y(3)*(G(7,5)+H(5,7))
1+AC(7)*(G(7,7)+H(7,7))
T61=F(1,1)*Y(4)+F(1,3)*Y(5)+F(1,5)*Y(6)+F(1,7)*BC(7)
T62=F(3,1)*Y(4)+F(3,3)*Y(5)+F(3,5)*Y(6)+F(3,7)*BC(7)
T63=F(5,1)*Y(4)+F(5,3)*Y(5)+F(5,5)*Y(6)+F(5,7)*BC(7)
T64=F(7,1)*Y(4)+F(7,3)*Y(5)+F(7,5)*Y(6)+F(7,7)*BC(7)
T65=F(1,1)*Y(1)+F(3,1)*Y(2)+F(5,1)*Y(3)+F(7,1)*AC(7)
T66=F(1,3)*Y(1)+F(3,3)*Y(2)+F(5,3)*Y(3)+F(7,3)*AC(7)
T67=F(1,5)*Y(1)+F(3,5)*Y(2)+F(5,5)*Y(3)+F(7,5)*AC(7)
T68=F(1,7)*Y(1)+F(3,7)*Y(2)+F(5,7)*Y(3)+F(7,7)*AC(7)

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GAM1=T11-64.0*T14
GAM2=T12-16.0*T14
GAM3=T13-4.0*T14
ALPH1=T51-64.0*T54
ALPH2=T52-16.0*T54
ALPH3=T53-4.0*T54
FT1=BETA1*ALPH3-BETA5*GAM3
FT2=GAM2*ALPH3-ALPH2*GAM3
FT3=GAM1*ALPH3-ALPH1*GAM3
Z(1)=(FT1*T32-FT2*BETA3)/(FT3*T32-FT2*T31)
Z(2)=(BETA3-T31*Z(1))/T32
Z(3)=(BETA1-GAM1*Z(1)-GAM2*Z(2))/GAM3
Z(4)=BETA2/T25
Z(5)=(BETA4-T41*Z(1)-T45*Z(4))/T46
Z(6)=((BETA6-64.0*T68)-(T61-64.0*T64)*Z(1)-(T62-16.0*T64)*Z(2)-
1(T63-4.0*T64)*Z(3)-(T65-64.0*T68)*Z(4)-(T66-16.0*T68)*Z(5))/
2(T67-4.0*T68)
DPDR=-Y(1)*Z(1)+Y(4)**2/R+2.0*Y(2)/RE
RETURN

```

C
C
C THIS SECTION IS FOR EIGHTH ORDER POLYNOMIALS FOR U AND V

300 CONTINUE

IF (KONT.EQ.1) GO TO 320

AC(1)=Y(1)

AC(3)=Y(2)

AC(5)=Y(3)

AC(7)=Y(4)

AC(9)=-256.*AC(1)-64.*AC(3)-16.*AC(5)-4.*AC(7)

BC(1)=Y(5)

BC(3)=Y(6)

BC(5)=Y(7)

BC(7)=Y(8)

BC(9)=256.0*(R-Y(5))-64.0*Y(6)-16.0*Y(7)-4.0*Y(8)

320 CONTINUE

BETA7=0.0

BETA8=0.0

DO 340 I=1,9,2

DO 340 J=1,9,2

BETA7=BETA7+H(I,J)*BC(I)*BC(J)-G(I,J)*AC(I)*AC(J)

BETA8=BETA8+F(I,J)*AC(I)*BC(J)

340 CONTINUE

BETA2=-Y(1)*Y(5)/R+2.0*Y(6)/RE

BETA3=12.0*Y(3)/RE+2.0/R*(Y(1)*Y(2)+Y(5)*Y(6))

BETA4=12.0*Y(7)/RE+(Y(1)*Y(6)-Y(2)*Y(5))/R

BETA5=2.0*Y(5)*Y(7)/R+2./3.*Y(2)**2/R+Y(6)**2/R+4.0*Y(1)*Y(3)/R
1+30.0*Y(4)/RE

BETA6=-Y(3)*Y(5)/R-Y(2)*Y(6)/(3.0*R)+3.0*Y(1)*Y(7)/R+30.0*Y(8)/RE

BETA7=BETA7/R+6.0*UO/(R*RE)+(AC(3)+AC(5)/2.0+3.0*AC(7)/16.0+
1AC(9)/16.0)/RE

BETA8=-2.0*BETA8/R+(BC(3)+BC(5)/2.0+3.0*BC(7)/16.0+BC(9)/16.0)/RE

T26=Y(1)

T31=-Y(2)

T32=Y(1)

T41=-2.0*Y(6)

T46=Y(2)
T47=Y(1)
T51=-3.0*Y(3)
T52=Y(2)/3.0
T53=Y(1)
T61=-4.0*Y(7)
T62=-2./3.*Y(6)
T66=Y(3)
T67=Y(2)
T68=Y(1)
T71=Y(1)*(G(1,1)+H(1,1))+Y(2)*(G(1,3)+H(3,1))+Y(3)*(G(1,5)+H(5,1))
1+AC(7)*(G(1,7)+H(7,1))+AC(9)*(G(1,9)+H(9,1))
T72=Y(1)*(G(3,1)+H(1,3))+Y(2)*(G(3,3)+H(3,3))+Y(3)*(G(3,5)+H(5,3))
1+AC(7)*(G(3,7)+H(7,3))+AC(9)*(G(3,9)+H(9,3))
T73=Y(1)*(G(5,1)+H(1,5))+Y(2)*(G(5,3)+H(3,5))+Y(3)*(G(5,5)+H(5,5))
1+AC(7)*(G(5,7)+H(7,5))+AC(9)*(G(5,9)+H(9,5))
T74=Y(1)*(G(7,1)+H(1,7))+Y(2)*(G(7,3)+H(3,7))+Y(3)*(G(7,5)+H(5,7))
1+AC(7)*(G(7,7)+H(7,7))+AC(9)*(G(7,9)+H(9,7))
T75=Y(1)*(G(9,1)+H(1,9))+Y(2)*(G(9,3)+H(3,9))+Y(3)*(G(9,5)+H(5,9))
1+AC(7)*(G(9,7)+H(7,9))+AC(9)*(G(9,9)+H(9,9))
T81=Y(5)*F(1,1)+Y(6)*F(1,3)+Y(7)*F(1,5)+Y(8)*F(1,7)+BC(9)*F(1,9)
T82=Y(5)*F(3,1)+Y(6)*F(3,3)+Y(7)*F(3,5)+Y(8)*F(3,7)+BC(9)*F(3,9)
T83=Y(5)*F(5,1)+Y(6)*F(5,3)+Y(7)*F(5,5)+Y(8)*F(5,7)+BC(9)*F(5,9)
T84=Y(5)*F(7,1)+Y(6)*F(7,3)+Y(7)*F(7,5)+Y(8)*F(7,7)+BC(9)*F(7,9)
T85=Y(5)*F(9,1)+Y(6)*F(9,3)+Y(7)*F(9,5)+Y(8)*F(9,7)+BC(9)*F(9,9)
T86=Y(1)*F(1,1)+Y(2)*F(3,1)+Y(3)*F(5,1)+Y(4)*F(7,1)+AC(9)*F(9,1)
T87=Y(1)*F(1,3)+Y(2)*F(3,3)+Y(3)*F(5,3)+Y(4)*F(7,3)+AC(9)*F(9,3)
T88=Y(1)*F(1,5)+Y(2)*F(3,5)+Y(3)*F(5,5)+Y(4)*F(7,5)+AC(9)*F(9,5)
T89=Y(1)*F(1,7)+Y(2)*F(3,7)+Y(3)*F(5,7)+Y(4)*F(7,7)+AC(9)*F(9,7)
T810=Y(1)*F(1,9)+Y(2)*F(3,9)+Y(3)*F(5,9)+Y(4)*F(7,9)+AC(9)*F(9,9)
ALPHA1=T11-256.0*T15
ALPHA2=T12-64.0*T15
ALPHA3=T13-16.0*T15
ALPHA4=T14-4.0*T15
GAM1=T71-256.0*T75
GAM2=T72-64.0*T75
GAM3=T73-16.0*T75
GAM4=T74-4.0*T75
RH01=ALPHA1*GAM4-ALPHA4*GAM1
RH02=ALPHA2*GAM4-ALPHA4*GAM2
RH03=ALPHA3*GAM4-ALPHA4*GAM3
EP1=RH01*T53-RH03*T51
EP2=RH02*T53-RH03*T52
Z(1)=(((BETA1*GAM4-ALPHA4*BETA7)*T53-RH03*BETA5)*T32-EP2*BETA3)/
1*(EP1*T32-EP2*T31)
Z(2)=(BETA3-T31*Z(1))/T32
Z(3)=(BETA5-T51*Z(1)-T52*Z(2))/T53
Z(4)=(BETA1-ALPHA1*Z(1)-ALPHA2*Z(2)-ALPHA3*Z(3))/ALPHA4
Z(5)=BETA2/T26
Z(6)=(BETA4-T41*Z(1)-T46*Z(5))/T47
Z(7)=(BETA6-T61*Z(1)-T62*Z(2)-T66*Z(5)-T67*Z(6))/T68
Z(8)=((BETA8-256.0*T810)-(T81-256.0*T85)*Z(1)-(T82-64.0*T85)*Z(2)
1-(T83-16.0*T85)*Z(3)-(T84-4.0*T85)*Z(4)-(T86-256.0*T810)*Z(5)
2-(T87-64.0*T810)*Z(6)-(T88-16.0*T810)*Z(7))/(T89-4.0*T810)
DPDR=Y(5)**2/R+2.0*Y(2)/RE-Y(1)*Z(1)

RETURN
END