CHAPTER VII

THEORY OF DISPLACEMENTS—MOTORS

CHAPTER VIII

THEORY OF RESONATORS

CHAPTER IX

HIGH-FREQUENCY CURRENTS

CHAPTER X

CHARGED LINES

CHAPTER XI

TRANSFORMERS

APPENDIX

A TREATISE ON WAVE TRANSMISSION OF POWER

CHAPTER I

INTRODUCTORY

ONE of the fundamental problems of mechanical engineering is that of transmitting energy found in nature, after suitable transformation, to some point at which it can be made available for performing useful work.

The methods of transmitting energy known and practised by engineers are broadly included in two classes: mechanical, including hydraulic, pneumatic and wire rope methods; and electrical methods.

The present volume deals with a new method by which the problem has been solved by the author.

All methods of transmitting power through liquids, known as hydraulic methods, as hitherto applied, depend on the continuous transmission of pressure through a liquid so that pressure generated at one end of the line is utilised at the other end. The liquid in this form of transmission merely acts as an incompressible flexible connecting-rod.

The known pneumatic methods involve a flow in the pipes always in one direction, pressure being generated at one end of the system and utilised at the other end, but in this case the elasticity of the air employed is sometimes taken advantage of in the power utilisers.

In the wire-rope methods, the motive power is, as it were, attached by a string, as near as possible inextensible, to the power utilisers; the system depends on the longitudinal motion of the wire as a whole.

In all these known methods of applying mechanical means to the transmission of power from one point to a distant point elasticity has no direct function and is generally avoided or ignored.

The author's system depends on the elasticity of the medium through which the energy is transmitted. The essential feature of the system is that the particles of the medium employed, whether solid, liquid, or gaseous, are in a state of vibration about a mean position.

According to the new system, energy is transmitted from one point to another, which may be at a considerable distance, by means of impressed periodic variations of pressure or tension producing longitudinal vibrations in solid, liquid, or gaseous columns. The energy is transmitted by periodic changes of pressure and volume in the longitudinal direction, and may be described as wave transmission of power, or mechanical wave transmission.

CHAPTER II

ELEMENTARY PHYSICAL PRINCIPLES

THERE are many instances in nature of transmission of energy by vibrations; wave motion may almost be said to be the natural method of transmitting energy.

Let us consider some known phenomena of vibrations of particles of matter.

The transmission of sound through air is due to a vibratory motion set up by the source in the surrounding air; each particle of air in the neighbourhood of the source is put into a state of vibration about a mean position.

A common method of producing sound is to cause an elastic diaphragm to vibrate, impressing its vibrations on the surrounding air. By isolating the air to which the vibrations are transmitted, as, for instance, by means of a speaking-tube, the sound can be directed and a given quantity of energy of vibration produced can thus be transmitted over great distances.

Consider what is taking place in the tube when the contained air is set in motion by a diaphragm in a plane normal to the axis of the tube and vibrated about a mean position.

The first movement of the diaphragm in the direction of the tube displaces some air along the tube; this displacement is resisted by the still air further along the tube, so that a zone of compressed air is produced in the immediate neighbourhood of the diaphragm. At the same time the moving diaphragm is giving velocity to the particles of air in its immediate neighbourhood, and these particles communicate their velocity to those beyond them, and thus any disturbance once produced by the diaphragm must travel forward along the tube. On the return movement of the diaphragm, the compressed air in its immediate neighbourhood, being elastic, expands, and we have then a zone of low-pressure air in contact with the diaphragm.

The continuing vibrations of the diaphragm produce alternate zones of high and low pressure, and the disturbances so produced travel forward along the tube until the whole of the air particles in the tube are in a state of vibration; it has been found that the zones of high and low pressure travel along the tube with a definite velocity of about 330 metres per second, this velocity varying slightly with the diameter of the tube.

In a similar manner sound energy travels through other elastic media. The velocity through water has been found to be about 1435 metres per second.

As hitherto employed for the transmission of power in hydraulic and telpherage systems, liquid and solid connections have been considered as movable *en bloc*, and for practical purposes incompressible and inextensible. Both liquid and solid columns, however, are elastic, and this property can be made use of to transmit energy by vibrations of the particles of matter of which they are built up. We will first consider the case of liquid columns.

Assume that we have 150 metres of wrought-iron steam-pipe, of 2·5 cm. diameter and 0·5 cm. thickness of metal, closed at one end and filled with water; and suppose a fluid-tight piston is forced into the pipe under a steady pressure of 35 kg. per sq. cm. If the liquid were incompressible the increase in volume of the containing pipe under the pressure would allow the piston to enter about 1·5 cm.

If the pipe were absolutely inexpansible the pressure would compress the water to an extent that would allow the piston to enter about 26 cm.

It is seen, therefore, that the compression of the water in a wrought-iron steam-pipe of the size considered is the chief factor in the changes of volume which take place under pressure, and that the expansion of the containing pipe is almost negligible.

On removing the pressure from the piston, the water will again expand to its original volume. With other liquids similar results will be obtained. Assume now that the pipe, instead of being closed rigidly at one end, is closed by a light floating piston held always in contact with the liquid column, but free to move with the liquid; assume further that the working piston, instead of being slowly pushed into the pipe, is connected to a rapidly rotating crank, so that it moves with a simple harmonic motion, and that in addition to the piston impulses a steady pressure acts on the liquid column at both ends. The only resistance to the movement of the piston is then the inertia of the liquid column, and if the column is short the liquid will move as a solid mass. If, however, the column is of considerable length, the motion of the layers of liquid nearer the working piston is resisted by the inertia of the more remote layers, and on the in-stroke of the piston the liquid in its neighbourhood will be compressed and its volume diminished; it follows that the motion of the layers of liquid remote from the piston will be less than that of layers nearer to it.

At any given speed of rotation of the crank there will be a point in the liquid column at which, on the completion of the in-stroke of the piston, no movement of the liquid has occurred. The liquid between this point and the piston will at this moment be in a state of compression varying from a maximum at the piston to zero.

At the moment of maximum velocity of the piston, the velocity of the layer of liquid in contact with it will necessarily be greater than the velocity of the more remote layers, and the kinetic energy of the layers nearer the piston will, therefore, be transmitted in the forward direction along the column. The energy expended by the piston in its forward stroke at the end of this stroke is present in the liquid column, partly in the form of potential energy due to the decreased volume of the liquid under compression and partly as kinetic energy.

On the return stroke of the piston, the compression of the layer of liquid in contact with it decreases, and expansion of the liquid takes place between the piston and the point in the column at which the pressure is a maximum. As the point of maximum pressure moves away from the piston at the commencement of the return stroke, the velocity of the layer of liquid in contact with the piston is reversed, while the pressure of this layer diminishes until the piston is at the end of its out-stroke. At the end of this out-stroke the layer of liquid in contact with the piston is instantaneously at rest.

As the crank continues rotating, there are thus impressed on the liquid column a series of impulses sending a series of changes of pressure and volume along the column, the particles of liquid each vibrating about a mean position.

The considerations dealt with above as regards vibrations in liquid columns apply also to solids; this may be shown by considering the case of a long helical spring, one end of which is subjected to periodic shocks in the longitudinal direction. At each shock the end of the spring will be compressed and will again expand when the impulse is removed; the effect of the impulse, however, will travel along the spring in the direction of the shock with a definite velocity. The inertia of the coils of the spring remote from the end provides the resistance necessary to compress the first coils, but on the removal of the impulse expansion takes place in both directions, so that the wave of pressure and displacement travels along the spring.

An example of this occurs in practice in the case of the recoil springs of heavy ordnance, in which it has been noticed that pulses in the movements of the gun take place, due to the zones of compression in the recoil springs produced by the sudden shock of firing.

Consider now a very long steel wire connected to a crank so that the end is given a simple harmonic motion in the longitudinal direction, and suppose that the tension given by the crank is superposed on a steady tension in the wire so that no part of the wire is ever in a state of longitudinal compression.

As the crank rotates the end of the wire will be subjected to alternate maximum and minimum tensions occurring periodically; under certain conditions the wire, being elastic, will not move *en bloc*, but the periodic changes of tension will produce periodic displacements of the particles of the wire in the longitudinal direction, each particle vibrating about a mean position as in the case of the fluid columns discussed above.

In the transmission of sound through air we have seen that a series of vibrations is imparted to the air particles, causing them to move about a mean position; and thus a series of waves of alternate compression and rarefication travels forward from the source. If these waves fall on a sensitive receiver, such as the drum of the human ear, the receiver is set in vibration and the sound is heard. This is, in fact, an example of the transmission of energy by mechanical wave motion. Similarly, sound is transmitted through liquids and solids.

In order that a receiver may be able to respond to the vibrations falling on it, certain conditions are essential. The part of the receiver which is to be put in motion must be capable of vibrating at the periodicity of the vibrations which fall on it.

In the case of the human ear very sensitive receivers are found, which are tuned to or capable of adapting themselves readily to vibrations of different periodicity within certain limits of frequency. When, however, we come to the problem of detecting vibrations by mechanical means, and still more so when it is desired to transmit power economically by means of these vibrations, it is necessary that the part moved should be designed so that it can respond to the particular periodicity of vibration by which the power is transmitted. It is further necessary, if the part moved has to perform useful work, that the work should be performed in such a manner that the ability of the receiver to vibrate in unison with the impressed vibrations is not interfered with.

Although in some cases in which energy has been heretofore transmitted by vibrations in matter—as, for instance, the case of a tuning-fork made to respond to sound waves of its own frequency—the question of the period of vibration of the receiver has been considered; in no case, up to the present, has the tuned receiver been adapted to the performance of work. For the transmission of power by mechanical wave motion it is therefore necessary to devise means by which the vibrations in the transmission line may be received and converted to use.

Let us now consider further the case of a rapidly rotating crank causing a piston to reciprocate at the end of a long pipe containing liquid. We have seen above that a series of zones of high pressure and compression of the liquid alternating with zones of low pressure and expansion of the liquid are produced, and that these zones travel forward along the pipe.

In Fig. 1 suppose the crank a to be rotating uniformly, causing the piston b to reciprocate in the pipe c, which is full of liquid. At each in-stroke of the piston a zone of high pressure is formed, and these zones of high pressure, shown by shading, travel along the pipe away from the piston; between every pair of high pressure zones is a zone of low pressure shown light in the figure. The pressure at any point in the pipe, therefore, will go through a series of values from a maximum to a minimum, and these values will repeat periodically. Let the line ox represent the value of the mean pressure, then, with the piston in the position illustrated, the instantaneous pressures at different points along the pipe may be represented by the ordinates of the sine curve $efg \dots k$. As the rotation of the crank is uniform, it will be evident that the distances between successive points of maximum pressure will be equal. This uniform distance along the pipe at which the values of the pressure are repeated is the wave length of the vibrating movement of the liquid.

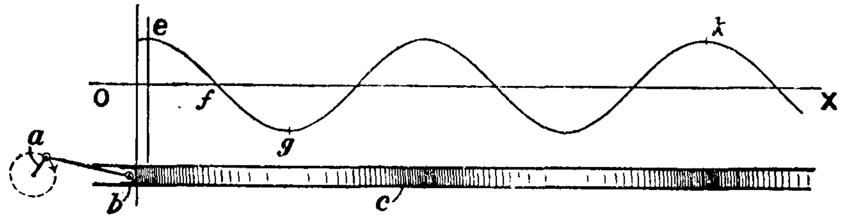

FIG. 1.

If v is the velocity with which these waves travel along the pipe, and n is the number of revolutions in unit time of the crank a, it will readily be seen that the wave length λ must be $\frac{v}{n}$.

Assume now that the pipe is of finite length and is closed at the point r at a distance from the piston b equal to an exact multiple of the wave length, and suppose that the stroke of the piston is small compared with the wave length as shown in Fig. 2.

The wave of compression will be stopped at r and reflected, and the reflected wave will travel back along the pipe.

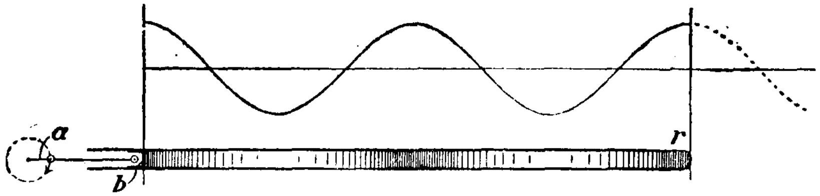

FIG. 2.

If the crank continues its rotation at uniform speed, with the length of pipe and speed of rotation we have taken—*i.e.*, with the distance from the piston b to the stop r an exact multiple of the wave length—a zone of maximum pressure will be just starting from the piston at the instant the reflected zone of maximum pressure reaches it; so that we shall have a wave of double the original amplitude travelling forward along the pipe. The next revolution of the crank will again add to the amplitude of the wave sent forward; and so on with successive revolutions. The result of this continual pouring in of energy is that the maximum pressure increases without limit till ultimately the pipe bursts.

It should be noticed that, in a wave of greater amplitude, the maximum pressures are increased, and the maximum velocities and distance of travel of the oscillating particles are also increased.

Suppose now that instead of closing the pipe rigidly at r we have at r a piston m connected to a crank n similar to a as shown in Fig. 3.

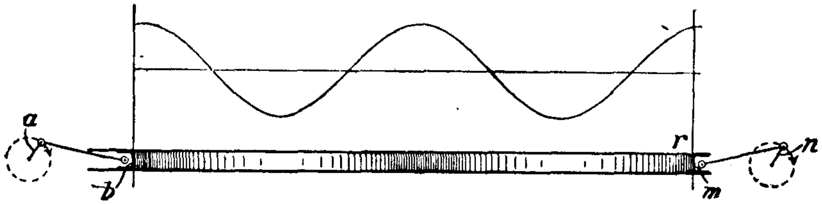

FIG. 3.

Suppose that the crank n is rotating at the same angular velocity and in the same phase as the crank a. If the liquid column were continued beyond the piston m, it is evident that the movement of the piston would produce in this column a series of waves which would be exactly similar to and a continuation of the waves between b and m.

The piston m, therefore, if moving synchronously with b, will be able to take up the whole energy of the waves produced by b and travelling along the pipe.

It will be seen, further, that the piston will be able to take up and utilise the whole of the energy of the waves travelling to it if placed at any point of the pipe, provided its time period of reciprocation is the same as that of the piston a, and provided that the phase of its movement is such as would produce a continuation beyond it of the impinging waves; that is to say, provided the piston movement is in phase with the movement of the layer of liquid in contact with it.

In the transmission of power by wave motion in this example, the maximum pressure in the pipe will at no point exceed the maximum pressure in the neighbourhood of the working piston, however long the transmission line may be; and will be the same whether the line is a single wave length or any number of wave lengths. Also the two pistons may be moving in the same or in opposite directions, and their motions may differ in phase by any angle according to the relation between the distance from one to the other and the wave length.

FIG. 4.

In the example above discussed, the whole of the energy put into the liquid column by the piston b can be taken up by the piston m. If more energy is put in by b than is taken up by the piston m, assuming no frictional losses, it is obvious that reflected waves must be formed as the direct waves fall on the piston m. The result of this will be that the surplus energy will remain in the liquid and the continuation of the rotation will continually pour in energy, increasing the maximum pressure indefinitely till ultimately, as in the case of the closed pipe, the pipe will burst.

Suppose that, in the case of a closed pipe having a length of several wave lengths, a vessel d completely filled with liquid, of considerable volume in proportion to the stroke volume of the piston b, and with rigid walls, is placed in communication with the pipe in the neighbourhood of the piston, as shown in Fig. 4. At each in-stroke of the piston a flow will take place through the entrance to the vessel d, and the liquid in this vessel will be compressed, and at each out-stroke of the piston the liquid in the vessel will again expand; according to the volume of the vessel more or less liquid will flow into it and out of it at each revolution of the crank. The capacity d will thus act as a spring, taking up the energy of the direct and reflected waves when the pressure is high, and giving back this energy when the pressure falls; the mean pressure in the vessel d and in the pipe will be the same, so that when the successive reflected waves in the pipe have been produced and have reached a certain amplitude equivalent to this mean pressure, the piston will merely exert energy in compressing the liquid in the vessel d on its in-stroke, and the liquid acting as a spring will restore this energy to the piston on its out-stroke. The result of this is that when the reflected waves have been produced, there will be a series of *stationary* waves in the pipe, and no further increase of energy in the liquid will take place and the pressures in the pipe will never exceed the fixed limit.

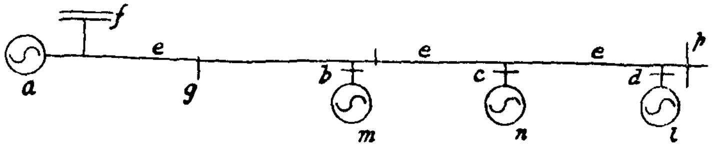

Fig. 5.

By using a vessel such as d, therefore, the pipe can be completely or partially closed. It is therefore possible to place at the far end or other point of the pipe apparatus for utilising only part of the energy of the wave, and the rotating crank a will only require to perform work to the extent of the energy actually utilised.

Consider now a case (Fig. 5) in which waves are transmitted by a reciprocating piston a along a line eee provided with branches. Assume that the pipe e is closed at p at a distance of one complete wave length from the wave generator a; and that there are branches bcd at the half, three-quarter and full wave-length distances respectively. We know from the cases discussed above that if the cock p is closed and the cock d opened, leading to a motor l rotating at the synchronous speed, the motor l will be able to take up the whole of the energy put into the liquid by the pump.

We also know that if all the cocks are closed stationary waves will be produced in the pipe e having maximum variations of pressure at the end p and at the half wave length b. At these points the flow will always be zero, while the pressure will alternate between maximum and minimum values determined by the capacity f, consisting of a closed vessel filled with liquid. At the quarter and three-quarter wave length g and c respectively the flow will alternate between maximum and minimum values, but the variation of pressure will remain zero.

In this case the points of maximum pressure and maximum movement do not travel along the pipe, but are fixed in position, and theoretically no energy flows from the generator. At the points of maximum movement no variation of pressure will occur; and at the points of maximum pressure variation there will be no movement of the liquid.

It is evident, therefore, that if the cock b leading directly to a motor m be opened, the motor m, running at the synchronous speed, will be able to take up all the energy given to the line. The stationary half wave between a and b will therefore disappear, its place being taken by the forward travelling wave, while between b and p the stationary wave will persist. If the cock c leading to the motor n at the three-quarter wave length be opened, all other cocks being closed, since at the point c the variation of pressure is always zero, no energy can be taken up by the motor, and the stationary wave will persist in the whole length of the pipe.

If the motor be connected at any intermediate point, part of the energy will be taken up by the motor, while the stationary wave will persist but will be of reduced amplitude between the generator a and the motor. The state of the liquid between the generator a and the motor may be considered as the resultant of two superposed waves: one a stationary wave and the other a travelling wave of flowing energy.

Assume now that the motor l is not capable of taking up all the energy which can be transmitted to the line by the generator a; then we shall have superposed in the pipe a system of stationary waves and a system of waves travelling along the pipe, so that there will be no point in the pipe at which the variation of pressure will always be zero, consequently a motor connected at any point of the pipe will be able to take up and utilise a portion of the energy which is transmitted to the line.

We see, therefore, that if we have a number of motors all connected to the line, every one of them will be able to take some energy and do some useful work. It is only when no energy is being utilised that points at which the variation of pressure is *permanently* zero can exist.

It is seen from the preceding discussion that periodic variations of pressure and volume can be impressed on columns of gases, liquids or solids; and can be made to travel along such columns, causing the several particles to be set in vibration about their mean positions.

It is further clear that under certain conditions the mechanical energy so transmitted can be made available for the performance of useful work.

The theoretical considerations on which such mechanical wave transmission depends are investigated analytically in the following chapters.

CHAPTER III

DEFINITIONS

Alternating Fluid Currents.—For any flow of fluid in full pipes, if

ω = the sectional area of the pipe in square centimetres,
v = the velocity of the fluid at any instant in centimetres per second,

and

i = the flow of liquid in cubic centimetres per second, we have,

$$i = v\omega.$$

Suppose that the current is produced by a piston moving in a cylinder of section Ω sq. cm. with a simple harmonic motion.

Let

r = the equivalent length of the driving crank in centimetres,
a = the angular velocity of the crank or the *pulsation* in radians per second,
n = the number of revolutions of the crank per second,

Then the flow from the cylinder to the pipe at any instant will be

$$i = I \sin (at + \phi) \quad \text{. . . . .} \quad (1)$$

where

$I = ra\,\Omega$ = the maximum alternating flow in cm.^{3}/sec., or the *amplitude* of the flow.
t = the time in seconds,
ϕ = the angle of phase,

and if

T = the period of one complete alternation, equal to the time of one complete revolution of the crank,

we have

$$a = 2\pi n,$$

$$n = \frac{1}{T}$$